2021 AIChE ANNUAL MEETING

NOVEMBER 7-11 • BOSTON, MA

NOVEMBER 15-19 • VIRTUAL

Linking Measurements and Statistical Methodology through the Characterization of Polymeric Materials: Hierarchical Analysis of Gel Permeation Chromatography Data

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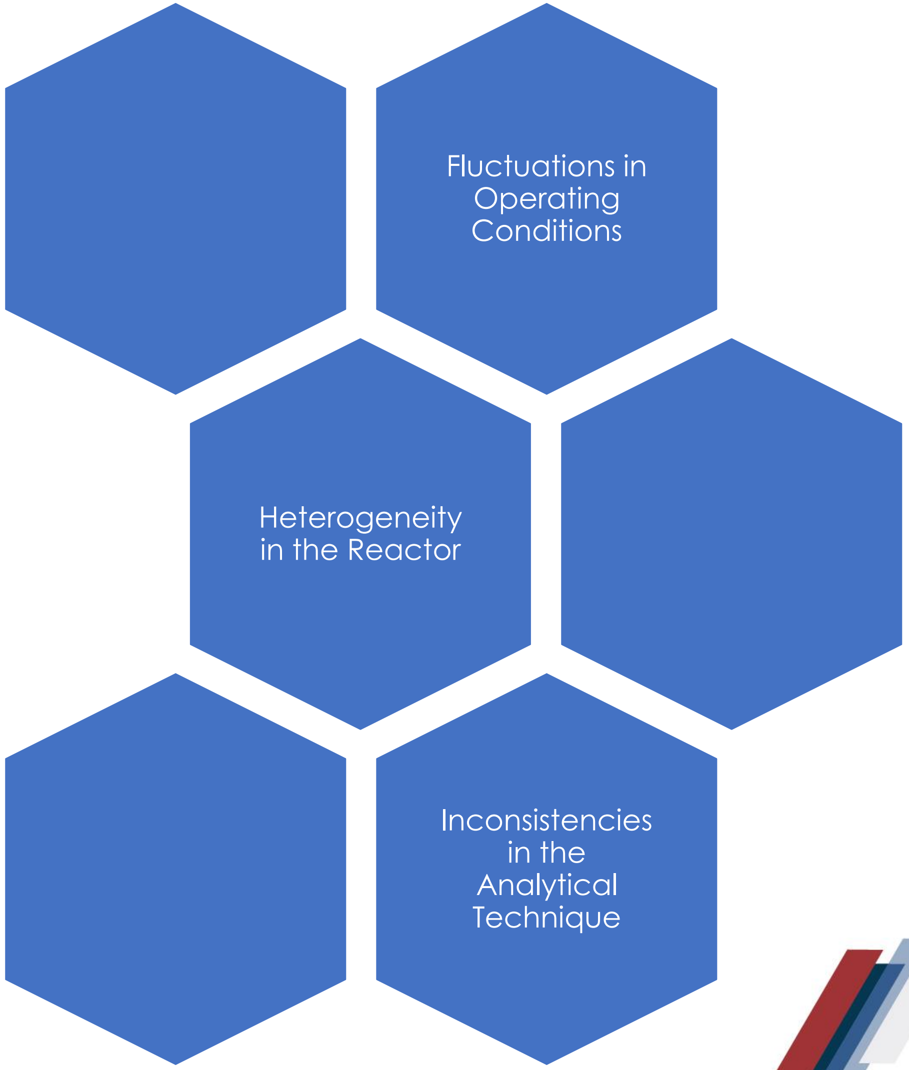
Building the Bridge in 21st Century Education:
Chemical Engineering Industry + Academia

Motivation

- Consider synthesizing some material and then analyzing the material using a property characterization technique in the lab…
  - If we independently replicate the synthesis and subsequent characterization several times, will we get the same result?

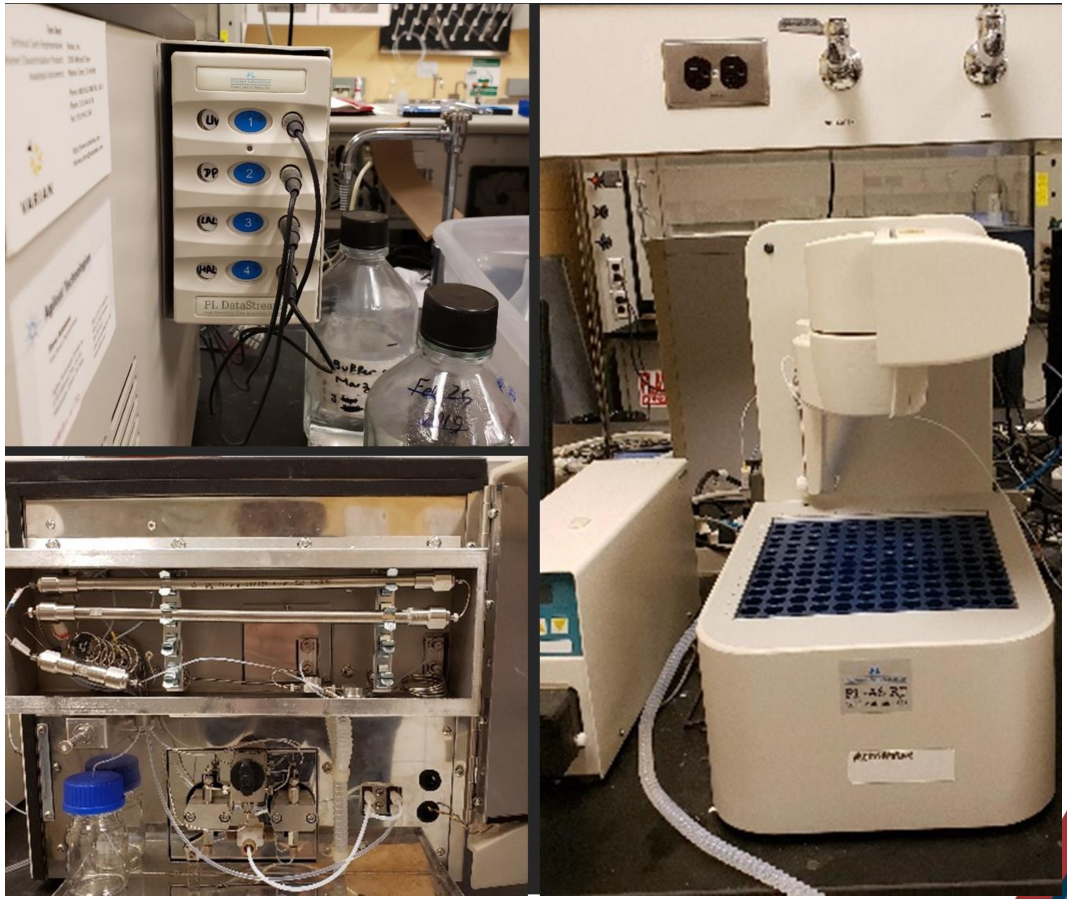


Hierarchical Design of Experiments

- For any process with multiple steps or stages, it can be useful to know whether the variance is equally a result of all operating stages, or if certain process steps are contributing most of the variance
  - **Hierarchical design of experiments:** variance decomposition technique where the overall variance is separated into several components…this allows us to identify and quantify different sources of variation within the data

Gel Permeation Chromatography

- Opportunity to introduce hierarchical design methodology to students using **characterization of polymeric materials**
    - Specifically, hierarchical analysis of aqueous gel permeation chromatography (GPC) data



Historical Background

- Dubé and Penlidis (1996). *Hierarchical data analysis of a replicate experiment in emulsion terpolymerization.* AIChE J. 42(7): 1985-1994.
- D'Agnillo, Soares and Penlidis (1999). *A hierarchical data analysis of a replicate experiment in polyethylene synthesis with high-temperature gel permeation chromatography.* Polym. React. Eng. 7(2): 259-281.
- Nabifar (2012) *Hierarchical Data Analysis of a Replicated NMRP of STY/DVB.* PhD Thesis.
- Filipovic, Scott and Penlidis (2021). *Hierarchical Data Analysis for the Characterization of Polymeric Materials: Linking Measurements and Statistical Methodology.* Chem. Eng. Ed. 55(1), 11-22.

Questions inspired by research provide a valuable opportunity to engage undergraduate students and to expose them to additional statistical tools.

Project Overview

- Molecular weight determination of water-soluble terpolymers for enhanced oil recovery (polymer flooding)
  - **Formulation:** prepare solutions with varying compositions of 2-acrylamido-2-methylpropane sulfonic acid (AMPS), acrylamide (AAm) and acrylic acid (AAc) in water
  - **Solution preparation:** add initiator and adjust pH, ionic strength, monomer concentration, etc.
  - **Synthesis:** separate pre-polymer solution into several vials (samples), add to hot water bath to initiate polymerization, isolate samples
  - **Characterization:** prepare samples for GPC and perform analysis (dissolve samples in mobile phase, filter samples, run GPC)

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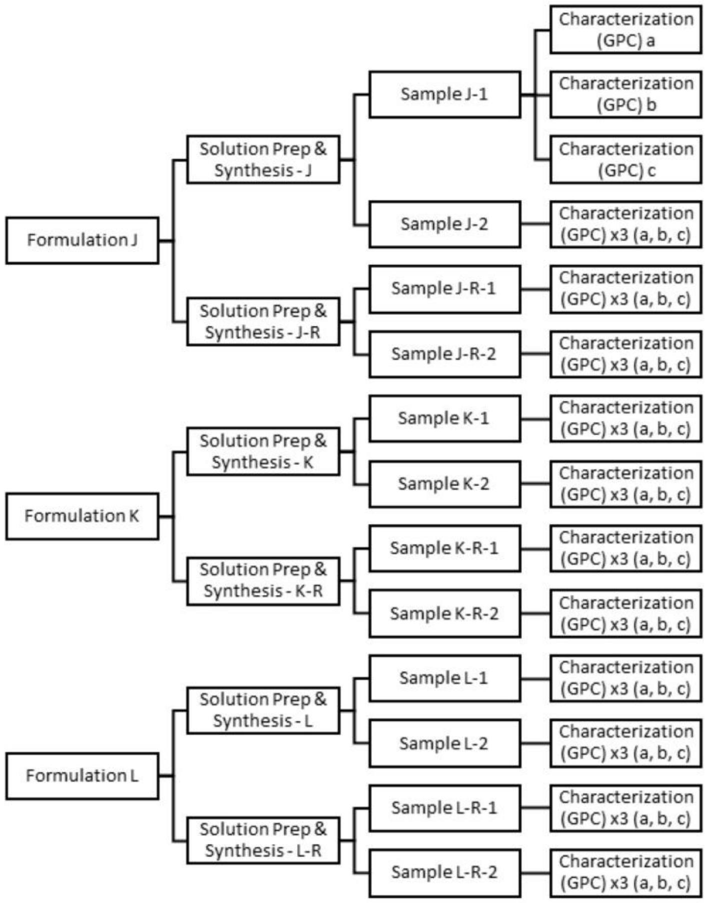
Project Overview



3 formulations

×2 solution preparation & synthesis replicates

×2 samples

×3 GPC characterization

= 36 GPC measurements (including $\bar{M}_w$, $\bar{M}_n$, M_p, PDI)

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Hierarchical Analysis

A: # formulation replicates
B: # solution replicates
C: # sample replicates
D: # GPC replicates

TABLE 1
Generalized ANOVA Table for a Nested Design with Four Levels

Source	Sum of Squares	Degrees of Freedom	MS	Expected Value of Mean Square (MS)	Component Variance Estimates
Average	$ABCD(\bar{y}^2)$	1			
Formulation	$BCD\sum_{a=1}^{A}(\bar{y}_a-\bar{y})^2$	A-1	m_A	$BCD\hat{\sigma}_A^2+CD\hat{\sigma}_B^2+D\hat{\sigma}_C^2+\hat{\sigma}_D^2$	$\hat{\sigma}_A^2=\frac{m_A-m_B}{BCD}$
Solution	$CD\sum_{a=1}^{A}\sum_{b=1}^{B}(\bar{y}_{ab}-\bar{y}_a)^2$	A(B-1)	m_B	$CD\hat{\sigma}_B^2+D\hat{\sigma}_C^2+\hat{\sigma}_D^2$	$\hat{\sigma}_B^2=\frac{m_B-m_C}{CD}$
Sample	$D\sum_{a=1}^{A}\sum_{b=1}^{B}\sum_{c=1}^{C}(\bar{y}_{abc}-\bar{y}_{ab})^2$	AB(C-1)	m_C	$D\hat{\sigma}_C^2+\hat{\sigma}_D^2$	$\hat{\sigma}_C^2=\frac{m_C-m_D}{D}$
GPC	$\sum_{a=1}^{A}\sum_{b=1}^{B}\sum_{c=1}^{C}\sum_{d=1}^{D}(y_{abcd}-\bar{y}_{abc})^2$	ABC(D-1)	m_D	$\hat{\sigma}_D^2$	$\hat{\sigma}_D^2=m_D$
Total	$\sum_{a=1}^{A}\sum_{b=1}^{B}\sum_{c=1}^{C}\sum_{d=1}^{D}(y_{abcd})^2$	ABCD			

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Hierarchical Analysis

TABLE 3 ANOVA Table for AMPS/AAm/AAc Study (A×B×C×D = 3x2x2x3)			
Source of Variation	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**
Average	9.11×10^{13}	1	
Formulation	5.03×10^{10}	2	2.51×10^{10}
Solution	3.26×10^{9}	3	1.09×10^{9}
Sample	3.47×10^{10}	6	5.79×10^{9}
GPC	1.52×10^{11}	24	6.35×10^{9}
Total	9.13×10^{13}	36	

TABLE 4 F-Testing Results for AMPS/AAm/AAc Study (A×B×C×D = 3x2x2x3)			
Type of Test	F_{obs}	F_{crit}	**Reject null?**
Sample/GPC	0.91	2.51	Fail to reject
Solution/Sample	0.19	4.76	Fail to reject
Formulation/Solution	23.13	9.55	Reject

Null hypothesis:

- component variance estimate at the higher level does not provide a significant contribution to the overall variability ($\hat{\sigma}_i{}^2 = 0$)

If the variance at a higher design level is significantly larger than the next lowest level, then the variance component at that upper design level provides a significant contribution to the overall variability.

In this case, significant differences in variances were detected only at the formulation level ($\hat{\sigma}_A{}^2 \neq 0$).

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Potential Extensions

- What if we eliminate one of the formulations? Is the formulation ‘level’ still contributing significantly to the overall variability?
- How ‘different’ would a solution/synthesis step need to be before we detect a significant contribution to the overall variability?

Benefits of Teaching Hierarchical Design

- Technical skills:
  - Exposure to advanced statistical design and analysis techniques
  - Experience identifying and quantifying sources of error
  - Improved understanding of polymerization techniques and material characterization (molecular weight distributions)
  - Development of laboratory skills including sample preparation, instrument operation and data collection

Benefits of Teaching Hierarchical Design

- Transferrable skills:
  - Collaboration with classmates and communication of results
  - Exposure to complex, open-ended problems
  - Real-world extensions to industrial problem-solving (designing experiments, identifying sources of error, troubleshooting, etc.)

Benefits of Teaching Hierarchical Design

- Versatility:
  - Senior undergraduate research project
  - Undergraduate laboratory course
  - Lecture-based statistics course
  - Graduate student exercise
- Easily adapted to diverse student backgrounds, laboratory capabilities, and course timelines
  - Analysis of published experimental data
  - Material synthesis and/or characterization (not just GPC)

Concluding Remarks

- Teaching hierarchical design methodology provides students with an opportunity to identify and quantify different sources of variation within a process
- Case study: terpolymers for enhanced oil recovery were synthesized and characterized via gel permeation chromatography
  - Additional studies from references of slide 5
- Projects have real-world relevance and can be used to achieve a wide range of learning outcomes

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