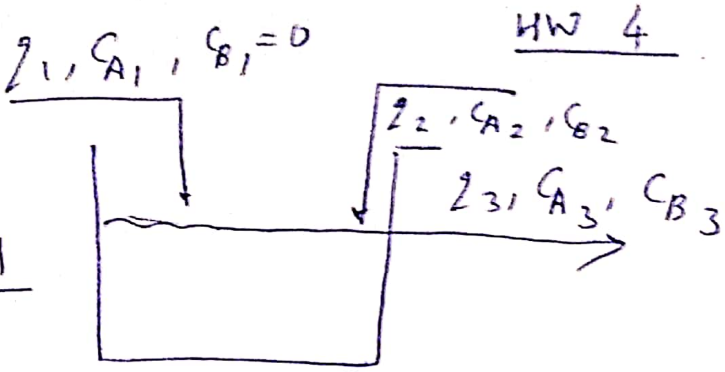
<u>HW 4</u>



Problem 1

$$A \underset{k_r}{\overset{k_f}{\rightleftharpoons}} B$$

$$r_f = k_f C_A$$

$$r_r = k_r C_B$$

$$0 = \frac{q_1}{V} C_{A_1} + \frac{q_2}{V} C_{A_2} - \left(k_f + \frac{q_1}{V} + \frac{q_2}{V}\right) C_{A_3} + k_r C_{B_3} \quad \text{——(1)}$$

$$0 = \frac{q_2}{V} C_{B_2} + k_f C_{A_3} - \left(k_r + \frac{q_1}{V} + \frac{q_2}{V}\right) C_{B_3} \quad \text{——(2)}$$

Substituting the values,

$$\Rightarrow 2C_{A_1} + C_{A_2} - 5C_{A_3} + C_{B_3} = 3 - 5C_{A_3} + C_{B_3} \quad \text{——(1)}$$

$$\Rightarrow C_{B_2} + 2C_{A_3} - 4C_{B_3} = 6 + 2C_{A_3} - 4C_{B_3} \quad \text{——(2)}$$

$$x = \begin{bmatrix} C_{A_3} \\ C_{B_3} \end{bmatrix} \Rightarrow \frac{dx}{dt} = \begin{bmatrix} -5 & 1 \\ 2 & -4 \end{bmatrix} x + \begin{bmatrix} 3 \\ 6 \end{bmatrix} = Ax + b$$

$$\frac{dx}{dt} = 0 \Rightarrow A\bar{x} + b = 0 \Rightarrow \bar{x} = A^{-1} b$$

$$A^{-1} = \frac{1}{det(A)} \begin{bmatrix} -4 & -1 \\ -2 & -5 \end{bmatrix} = -\frac{1}{18} \begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix}$$

$$\bar{x} = +\frac{1}{18} \begin{bmatrix} 4 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 18 \\ 36 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Problem 2)

$$2A \longrightarrow B$$
$$B \longrightarrow C$$
$$2B \longrightarrow D$$
$$2C + D \longrightarrow E$$

Given $r_1 = 2$

$$\begin{bmatrix} -1 & -2 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} -r_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} = Ar = b$$

$$\det(A) = 0 + 0 + 0 - \cancel{8}\, 2 - 2 - 0 = -4 \neq 0$$

Gaussian solution exists

$$A^{-1} = \begin{bmatrix} -0.5 & 0.5 & -1 \\ -0.25 & -0.25 & 0.5 \\ -0.25 & -0.25 & -0.5 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} -1 & -2 & 0 \\ 1 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} -0.5 & 0.5 & -1 \\ -0.25 & -0.25 & 0.5 \\ -0.25 & -0.25 & -0.5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} r_2 \\ r_3 \\ r_4 \end{bmatrix} = A^{-1} b = \begin{bmatrix} -0.5 & 0.5 & -1 \\ -0.25 & -0.25 & 0.5 \\ -0.25 & -0.25 & -0.5 \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 0.5 \end{bmatrix} \checkmark$$