Midterm Exam #1
ChE 231
Spring 2019

(1) (a) $p(A) = 0.99,\ n = 20$

$f(x=n) = p^n = (0.99)^{20} = 0.8179$

$f(x=0) = q^n = (1-p)^n = (0.01)^{20} = 10^{-40}$

(b) $f(x \geq 18) = f(x=18) + f(x=19) + f(x=20)$

$= \frac{20!}{18!(20-18)!}(0.99)^{18}(0.01)^2 + \frac{20!}{19!(20-19)!}(0.99)^{19}(0.01)^1$

$+ (0.99)^{20} = 0.0159 + 0.1652 + 0.8179 = 0.9990$

(2) (a) $x = [3.4\ \ 4.5\ \ 2.9\ \ 3.7\ \ 4.1\ \ 3.8\ \ 5.1\ \ 3.8],\ n = 8$

$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = 3.9125 \qquad s^2 = \frac{1}{8-1}\sum_{i=1}^{8}(x_i - \bar{x})^2 = 0.4498$

$s = \sqrt{s^2} = 0.6707 \Rightarrow [\bar{x}-3s\ \ \bar{x}+3s] = [1.900\ \ 9.614]$

x_4 is contained within limits $\Rightarrow$ not an outlier

(b) $\gamma = 0.90,\ n = 8-1 = 7,\ F(c) = \frac{1}{2}(1+\gamma) = 0.95 \Rightarrow c = 1.89$

$k = \frac{cs}{\sqrt{n}} = \frac{(1.89)(0.6707)}{\sqrt{8}} = 0.4482$

$CONF_{0.90}\ \{3.464 \leq \mu \leq 4.361\}$

(c) $\gamma = 0.90, n = 7, \quad F(c_1) = \frac{1}{2}(1-\gamma) = 0.05 \Rightarrow c_1 = 2.17$

$F(c_2) = \frac{1}{2}(1+\gamma) = 0.95 \Rightarrow c_2 = 14.07$

$k_1 = \frac{(n-1)s^2}{c_1} = \frac{(8-1)(0.4458)}{2.17} = 1.451$

$k_2 = \frac{(n-1)s^2}{c_2} = \frac{(8-1)(0.4458)}{14.07} = 0.2218$

$\Rightarrow \text{CONF}_{0.90}\{0.214 \le \sigma^2 \le 1.451\}$

(d) Hypothesis: $\mu = \mu_0 = 3$ vs. alternative $\mu = \mu_1 > \mu_0$

$\alpha = 0.10, n = 7, \quad P(T \le c) = \alpha = 0.90 \Rightarrow c = 1.41$

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{3.9125 - 3}{0.6707/\sqrt{8}} = 3.848 \Rightarrow t > c \Rightarrow$ reject hypothesis

(3)

Ext	1	2	3	4	5	6	7
x	-3	-2	-1	0	1	2	3
y	-1.1	0.2	-0.1	-0.3	1.4	0.6	0.8

$n = 7$

$\alpha = 0.05, \quad m = n-2 = 5, \quad P(T \le c) = 1-\alpha = 0.95 \Rightarrow c = 2.02$

$s_x^2 = \frac{1}{7-1}\sum_{i=1}^{7}(x_i - \bar{x})^2 = 4.667 \qquad \bar{x} = \frac{1}{7}\sum_{i=1}^{7} x_i = 0$

$s_y^2 = \frac{1}{7-1}\sum_{i=1}^{7}(y_i - \bar{y})^2 = 0.665 \qquad \bar{y} = \frac{1}{7}\sum_{i=1}^{7} y_i = 0.2143$

$$S_{xy} = \frac{1}{7-1}\sum_{i=1}^{7}(x_i-\bar{x})(y_i-\bar{y}) = 1.333$$

$$r = \frac{S_{xy}}{S_x S_y} = \frac{1.333}{\sqrt{4.667}\sqrt{0.665}} = 0.757$$

$$t = r\sqrt{\frac{n-2}{1-r^2}} = (0.757)\sqrt{\frac{7-2}{1-(0.757)^2}} = 2.591$$

$t > 1 \Rightarrow$ X & Y are correlated

(4) (a)

Expt	1	2	3	4	5	6	7
X	-3	-2	-1	0	1	2	3
Y	2.5	2.7	1.7	-1.6	-0.5	-2.5	-1.9

$$\bar{x} = 0, \quad \bar{y} = \frac{1}{7}\sum_{i=1}^{7} y_i = 0.0571$$

$$S_x^2 = 4.667 \qquad S_y^2 = \frac{1}{7-1}\sum_{i=1}^{7}(y_i-\bar{y})^2 = 4.846$$

$$S_{xy} = \frac{1}{7-1}\sum_{i=1}^{7}(x_i-\bar{x})(y_i-\bar{y}) = -4.300$$

$$y = k_1 x + k_0 \qquad k_1 = \frac{S_{xy}}{S_x^2} = \frac{-4.300}{4.667} = -0.9214$$

$$k_0 = \bar{x} - k_1\bar{x} = 0.0571 - (-0.9214)(0) = 0.0571$$

(b) $\alpha = 0.90,\ m = n-2 = 5,\ F(c) = \frac{1}{2}(1+\delta) = 0.95 \Rightarrow c = 2.02$

$$q_0 = (n-1)(S_y^2 - k_1^2 S_x^2) = (7-1)\left[4.846 - (-0.9214)^2(4.667)\right] = 5.304$$

$$K = c\sqrt{\frac{q_0}{(4-2)(4-1)\,S_X^2}} = (2.02)\sqrt{\frac{5.304}{(7-2)(7-1)(4.667)}}$$

$$= 0.393$$

$$CONF_{0.90}\{-1.315 \leq K_1 \leq -0.528\}$$