Assignment 6
Solutions

Problem 1

$$A=\begin{bmatrix}1 & 1 & 0\\ 0 & 2 & 0\\ 0 & -1 & 4\end{bmatrix}$$

1)

$$\det(A-\lambda I)=\begin{vmatrix}1-\lambda & 1 & 0\\ 0 & 2-\lambda & 0\\ 0 & -1 & 4-\lambda\end{vmatrix}=(1-\lambda)(2-\lambda)(4-\lambda)=0$$

$$\Rightarrow \lambda_1=1,\ \lambda_2=2,\ \lambda_3=4$$

2)

$$AX=\lambda X \Rightarrow (A-\lambda I)X=0 \Rightarrow \begin{aligned}(1-\lambda)X_1+X_2&=0\\ (2-\lambda)X_2&=0\\ -X_2+(4-\lambda)X_3&=0\end{aligned}$$

$\lambda_1=1$:
$$\begin{aligned}X_2&=0\\ X_2&=0\\ -X_2+3X_3&=0\end{aligned} \Rightarrow X^{(1)}=\begin{bmatrix}1\\0\\0\end{bmatrix}$$

$\lambda_2=2$:
$$\begin{aligned}-X_1+X_2&=0\\ 0&=0\\ -X_2+2X_3&=0\end{aligned} \Rightarrow X^{(2)}=\begin{bmatrix}2\\2\\1\end{bmatrix}$$

$\lambda_3=4$:
$$\begin{aligned}-3X_1+X_2&=0\\ -2X_2&=0\\ -X_2&=0\end{aligned} \Rightarrow X^{(3)}=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

3)

$$X=\begin{bmatrix}1 & 2 & 0\\ 0 & 2 & 0\\ 0 & 1 & 1\end{bmatrix}=\begin{bmatrix}X^{(1)} & X^{(2)} & X^{(3)}\end{bmatrix} \qquad D=X^{-1}AX$$

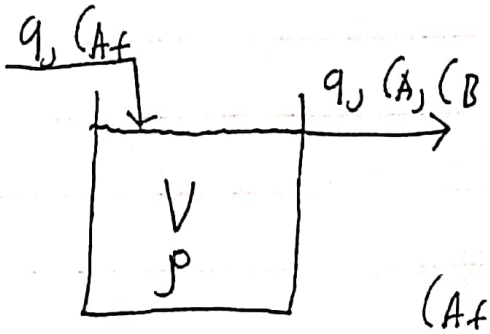
$$[X\ I] = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{2} & 1 \end{bmatrix} = [I\ X^{-1}]$$

$$D = X^{-1}AX = \begin{bmatrix} 1 & -1 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 0 \\ 0 & 4 & 0 \\ 0 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix} \checkmark$$

Problem 2




$r_A = -k C_A$

$A \rightarrow B$

$C_{Af} = 1, \quad V = 1$

| Expt | 1 | 2 | 3 |
|---|---|---|---|
| $\bar{q}$ | 1/2 | 1 | 2 |
| $\bar{C}_A$ | 0.35 | 0.45 | 0.70 |

1)

$$V \frac{dC_A}{dt} = q(C_{Af} - C_A) - k C_A V$$

$$0 = \bar{q}(C_{Af} - \bar{C}_A) - k \bar{C}_A V$$

Expt

$$\begin{matrix}1\\2\\3\end{matrix} \begin{bmatrix}0\\0\\0\end{bmatrix} = \begin{bmatrix}0.325\\0.550\\0.600\end{bmatrix} - \begin{bmatrix}0.350\\0.450\\0.700\end{bmatrix} k \quad k = b - Ax \Rightarrow Ax = b$$

1 unknown, 3 equations $\Rightarrow$ overdetermined linear system

Least-squares problem: $\min_x (Ax-b)^T (Ax-b)$

$$\Rightarrow x = (A^T A)^{-1} A^T b$$

$$= \left( \begin{bmatrix}0.350 & 0.450 & 0.700\end{bmatrix} \begin{bmatrix}0.350\\0.450\\0.700\end{bmatrix} \right)^{-1} \begin{bmatrix}0.350 & 0.450 & 0.700\end{bmatrix} \begin{bmatrix}0.325\\0.550\\0.600\end{bmatrix}$$

$$X = k = \frac{1}{0.815}(0.781) = 0.96$$

2)

Individual expts

Expt 1: $k = \frac{0.325}{0.350} = 0.93$

2: $k = \frac{0.350}{0.450} = 1.22$

3: $k = \frac{0.600}{0.700} = 0.86$

Average $k = 1.00 \neq 0.96$