Final Exam
ChE 231
Spring 2019

Problem 1 (45 pts). Consider a continuous stirred tank used to dissolve a gas component into a liquid. Mass balances on the gas and liquid phases yield the following linear ODE system,

$$\begin{aligned}\frac{dC_g}{dt} &= \frac{q_g}{V_g}(C_{g,f} - C_g) - k_L a(C^* - C_L)\\ \frac{dC_L}{dt} &= -\frac{q_L}{V_L}C_L + k_L a(C^* - C_L)\end{aligned}$$

where C_g and C_L are molar concentrations of the component in the gas and liquid phases, q_g and q_L are volumetric flow rates of the gas and liquid phases, V_g and V_L are the volumes of the gas and liquid phases, $C_{g,f}$ is the molar concentration of the gas component fed to the tank, and $k_L a$ is the gas-liqud mass transfer coefficient. The saturated liquid concentration of the component is calculated as $C^* = HC_g$, where H is the Henry's law constant.

1. (5 points) Given the parameter values $\frac{q_g}{V_g} = 2$, $\frac{q_L}{V_L} = 1$, $k_L a = 1$, $H = 0.5$ and $C_{g,f} = 4.5$ show that the ODE system can be written as,

$$\frac{d}{dt}\begin{bmatrix} C_g \\ C_L \end{bmatrix} = \begin{bmatrix} -2.5 & 1 \\ 0.5 & -2 \end{bmatrix}\begin{bmatrix} C_g \\ C_L \end{bmatrix} + \begin{bmatrix} 9 \\ 0 \end{bmatrix} = \mathbf{A}\begin{bmatrix} C_g \\ C_L \end{bmatrix} + \mathbf{b}$$

2. (10 points) Show that the ODE system has the steady-state solution $\bar{C}_g = 4$ and $\bar{C}_L = 1$ and express the ODE system in the form $\frac{d\mathbf{y}'}{dt} = \mathbf{A}\mathbf{y}'$

3. (15 points) Show that the eigenvalues and eigenvectors of the $\mathbf{A}$ matrix are,

$$\lambda_1 = -1.5, \qquad \mathbf{x}^{(1)}) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad \lambda_2 = -3, \qquad \mathbf{x}^{(2)}) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

4. (15 points) Given the initial conditions $C_g(0) = 4.5$ and $C_L(0) = 0$, find the solutions $C_g(t)$ and $C_L(t)$. Sketch the solutions.

Problem 2 (35 pts). Consider a continuous stirred tank bioreactor used to produce an microbial enzyme. Mass balances on the cellular biomass and secreted enzyme yield the following nonlinear ODE system,

$$
\begin{aligned}
\frac{dX}{dt} &= -DX + \mu(P)X = f_1(X,P) \\
\frac{dP}{dt} &= -DP - k_d P + vX = f_2(X,P)
\end{aligned}
$$

where X is the biomass concentration, P is the enzyme concentration, D is the dilution rate, k_d is the enzyme degradation rate, and v is the enzyme synthesis rate. The cellular growth rate μ is inhibited by enzyme $\mu(P) = \mu_m(1 - P/P_m)$ where μ_m is the maximum growth rate and P_m is the maximum enzyme concentration.

1. (10 points) Given the parameter values $D = 0.1$, $\mu_m = 0.2$, $v = 0.06$, $k_d = 0.02$ and $P_m = 10$, show that the ODE system has the steady state $\bar{X} = 10$ and $\bar{P} = 5$.

2. (15 points) Show that linearization of the ODE system at the steady state yields the linear ODE system,

$$
\frac{d}{dt}\begin{bmatrix} X' \\ P' \end{bmatrix} = \begin{bmatrix} 0 & -0.2 \\ 0.06 & -0.12 \end{bmatrix}\begin{bmatrix} X' \\ P' \end{bmatrix} = \mathbf{A}\begin{bmatrix} X' \\ P' \end{bmatrix}
$$

3. (10 points) Calculate the eigenvalues of the $\mathbf{A}$ matrix and determine if the dynamic response is stable/unstable and oscillatory/non-oscillatory.

Problem 3 (20 pts) The bioreactor considered in Problem 2 was used to generate the following data for the effect of the dilution rate D on the steady-state biomass concentration X,

Experiment	1	2	3	4	5	6
D	0.04	0.05	0.06	0.07	0.08	0.09
X	8.00	8.75	9.33	9.75	10.00	10.08

1. (10 points) Perform linear regression analysis based on statistics to find the slope and intercept of the linear model $X = k_1 D + k_0$.

2. (10 points) Perform linear regression analysis based on the least-squares method to show that the two methods yield the same linear model.