Written Homework #3
ChE 231
Spring 2019

Problem 1. Consider a continuous reactor for producing a polymer product. The following data was obtained for the scaled polymer polydispersity of eight separate runs: $x = \{1.21\ 0.89\ 1.06\ 1.12\ 0.92\ 1.18\ 0.98\ 0.82\}$.

1. Compute the mean and variance from these samples. Compute the 99% confidence intervals on the mean and variance.

$$x = \{1.21\ 0.89\ 1.06\ 1.12\ 0.92\ 1.18\ 0.98\ 0.82\}.$$

$$\bar{x} = \frac{1}{8}\sum_{i=1}^{8} x_i = 1.023 \qquad s^2 = \frac{1}{7}\sum_{i=1}^{8}(x_i - \bar{x})^2 = 0.020$$

$$\gamma = 0.99,\ m = 8-1 = 7 \qquad F(c) = \frac{1}{2}(1+0.99) = 0.995 \Rightarrow c = 3.50$$

$$k = \frac{cs}{\sqrt{n}} = \frac{(3.50)\sqrt{0.020}}{\sqrt{8}} = 0.176$$

$$\text{CONF}_{0.99}\{\bar{x} - k \le \mu \le \bar{x} + k\} = \text{CONF}_{0.99}\{0.847 \le \mu \le 1.199\}$$

$$\gamma = 0.99,\ m = 7,\ F(c_1) = \frac{1}{2}(1 - 0.995) = 0.005 \Rightarrow c_1 = 0.99$$

$$F(c_2) = \frac{1}{2}(1 + 0.995) = 0.995 \Rightarrow c_2 = 20.28$$

$$k_1 = \frac{(8-1)(0.020)}{0.99} = 0.143 \qquad k_2 = \frac{(8-1)(0.020)}{20.28} = 0.007$$

$$\text{CONF}_{0.99}\{0.007 \le \sigma^2 \le 0.143\}$$

2. Test the hypothesis that the mean $\mu = \mu_0 = 1$ versus the alternative that $\mu = \mu_1 < \mu_0$ at a significance level $\alpha = 0.05$.

Hypothesis: $\mu = \mu_0 = 1$ Alternative: $\mu = \mu_1 < \mu_0$

$\alpha = 0.05$, $m = 8 - 1 = 7$ $P(T \le \tau)_{\mu_0} = 1 - 0.05 = 0.95 \Rightarrow \tau = 1.89$

$P(T < c)_{\mu_0} = 0.05 \Rightarrow c = -\tau = -1.89$

$t = \dfrac{1.023 - 1}{\sqrt{0.020}/\sqrt{8}} = 0.46 > c = -1.89 \Rightarrow$ accept hypothesis

3. Test the hypothesis that the variance $\sigma^2 = \sigma_0^2 = 0.015$ versus the alternative that $\sigma^2 = \sigma_1^2 > \sigma_0^2$ at a significance level $\alpha = 0.05$.

Hypothesis: $\sigma^2 = \sigma_0^2 = 0.015$, Alternative: $\sigma^2 = \sigma_1^2 > \sigma_0^2$

$\alpha = 0.05$, $m = 7$ $P(Y \le c) = 1 - 0.05 = 0.95 \Rightarrow c = 14.07$

$c^* = \dfrac{(0.015)(14.07)}{8 - 1} = 0.030$

$s^2 = 0.020 < c^* = 0.030 \Rightarrow$ accept hypothesis

Problem 2. Consider a system for manufacturing spherical nanoparticles. The following diameter measurements were obtained for 5 nanoparticles: $x = \{0.12\ 0.09\ 0.11\ 0.10\ 0.08\}$

1. Compute the mean and variance from these samples. Compute the 90% confidence intervals on the mean and variance.

$$X = \{0.12 \quad 0.09 \quad 0.11 \quad 0.10 \quad 0.08\}$$

$$\bar{X} = \frac{1}{5}\sum_{j=1}^{5} X_i = 0.10 \qquad s^2 = \frac{1}{5-1}\sum_{j=1}^{5}(x_i - \bar{x})^2 = 2.5 \times 10^{-4}$$

Mean: $\gamma = 0.90,\ n = 5-1 = 4,\ F(c) = \frac{1}{2}(1+\gamma) = 0.95 \Rightarrow c = 2.13$

$$k = \frac{c\sqrt{s^2}}{\sqrt{n}} = \frac{(2.13)\sqrt{2.5 \times 10^{-4}}}{\sqrt{5}} = 0.015$$

$$\mathrm{CONF}_{0.90}\{0.085 \leq \mu \leq 0.115\}$$

Variance: $\gamma = 0.90,\ m = 4,\ F(c_1) = \frac{1}{2}(1-\gamma) = 0.05,\ c_1 = 0.71$

$F(c_2) = \frac{1}{2}(1+\gamma) = 0.95,\ c_2 = 9.49$

$$k_1 = \frac{(n-1)s^2}{c_1} = \frac{(4)(2.5 \times 10^{-4})}{0.71} = 1.40 \times 10^{-3}$$

$$k_2 = \frac{(n-1)s^2}{c_2} = \frac{(4)(2.5 \times 10^{-4})}{9.49} = 1.05 \times 10^{-4}$$

$$\mathrm{CONF}_{0.90}\{1.05 \times 10^{-4} \leq \sigma^2 \leq 1.40 \times 10^{-3}\}$$

2. (5 pts) Test the hypothesis that the mean $\mu = \mu_0 = 0.10$ versus the alternative that $\mu < \mu_0$ at a significance level $\alpha = 0.05$. Determine the smallest value $\mu_0 > 0.10$ such that the hypothesis would be rejected.

Hypothesis: $\mu_0 = \mu_0 = 0.10$ Alternative: $\mu < \mu_0$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = 0 \qquad P(T < c)_{\mu_0} = \alpha = 0.05$$

$$m = n - 1 = 4 \qquad \Rightarrow c = -2.13$$

$t > c \Rightarrow$ accept hypothesis

$$t = c \Rightarrow \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = c \Rightarrow \mu_0 = \bar{x} - c\frac{s}{\sqrt{n}} =$$

$$= 0.10 - \frac{\sqrt{2.5 \times 10^{-4}}}{\sqrt{5}}(-2.13) = 0.115$$

So reject hypothesis if $\mu_0 > 0.115$