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```
 clear; clc;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Solution for ChE 231 MATLAB Assignment 3 %
%                 Spring 2019               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Given Parameters

Define given parameters

```
Y_XS=0.5;    %g/g
alpha=2.8;   %g/g
beta=0.12;   %/hr
mu_m=0.75;   %/hr
Pm=60;       %g/L
Km=2.0;      %g/L
Ki=23;       %g/L
D=0.25;      %/hr
Sf=10;       %g/L
p=[Y_XS alpha beta mu_m Pm Km Ki D Sf];

% % Plot differential equations at various concentrations
% % in order to visually find range around each steady state
% X_vec=0:0.1:4; S_vec=0:0.1:2; P_vec=0:1:5;
%
% for k=1:length(P_vec)
%     for j=1:length(S_vec)
%         for i=1:length(X_vec)
%             X=X_vec(i);
%             S=S_vec(j);
%             P=P_vec(k);
%             F=bioreactor([X,S,P],p);
%             dX(i,j)=F(1);
%             dS(i,j)=F(2);
%             dP(i,j)=F(3);
%         end
%     end
%     figure(k)
%     mesh(X_vec,S_vec,dX','k')
```

```matlab
%       hold on
%       mesh(X_vec,S_vec,dS')
%       mesh(X_vec,S_vec,dP')
%       legend('dX','dS','dP')
%       xlabel('X'); ylabel('S'); zlabel('differential');
%       clear dX dS dP
% end
```

Question 1

```matlab
display('---------------------------------------------')
display('Question 1:')

% solves washout steady state
x0=[0 100 0];
options=optimoptions('fsolve','display','off');
F_SS1=fsolve(@(x) bioreactor(x,p),x0,options);
display(F_SS1)

% solves non-trivial steady state
x0=[1 50 50];
options=optimoptions('fsolve','display','off');
F_SS2=fsolve(@(x) bioreactor(x,p),x0,options);
display(F_SS2)
display('---------------------------------------------')
```

```
---------------------------------------------
Question 1:

F_SS1 =

   -0.0000    10.0000    -0.0000


F_SS2 =

    4.1949    1.6103    13.7592

---------------------------------------------
```

Question 2

```matlab
display('Question 2:')
display('Linearization dx/dt=Ax, where x=[X S P]''':')
syms X S P
f=bioreactor([X S P],p);
A=jacobian(f,[X S P]);
X=F_SS2(1); S=F_SS2(2); P=F_SS2(3);
A=subs(A);
A=double(A);
display(A);
display('Eigen values of A:')
lambda=eig(A);
```

```matlab
display(lambda)
display('---------------------------------------------')
```

Question 2:
Linearization dx/dt=Ax, where x=[X S P]':

A =

   -0.0000   0.3301   -0.0227
   -0.5000   -0.9103   0.0454
    0.8200   0.9244   -0.3135

Eigen values of A:

lambda =

   -0.7180
   -0.2558
   -0.2500

Question 3

```matlab
display('Question 3:')
display('(See MATLAB code for implementation of ode45 function)')
tspan=[0 10];
[t,sol]=ode45(@(t,x) bioreactor(x,p),tspan,F_SS2);

display('---------------------------------------------')
```
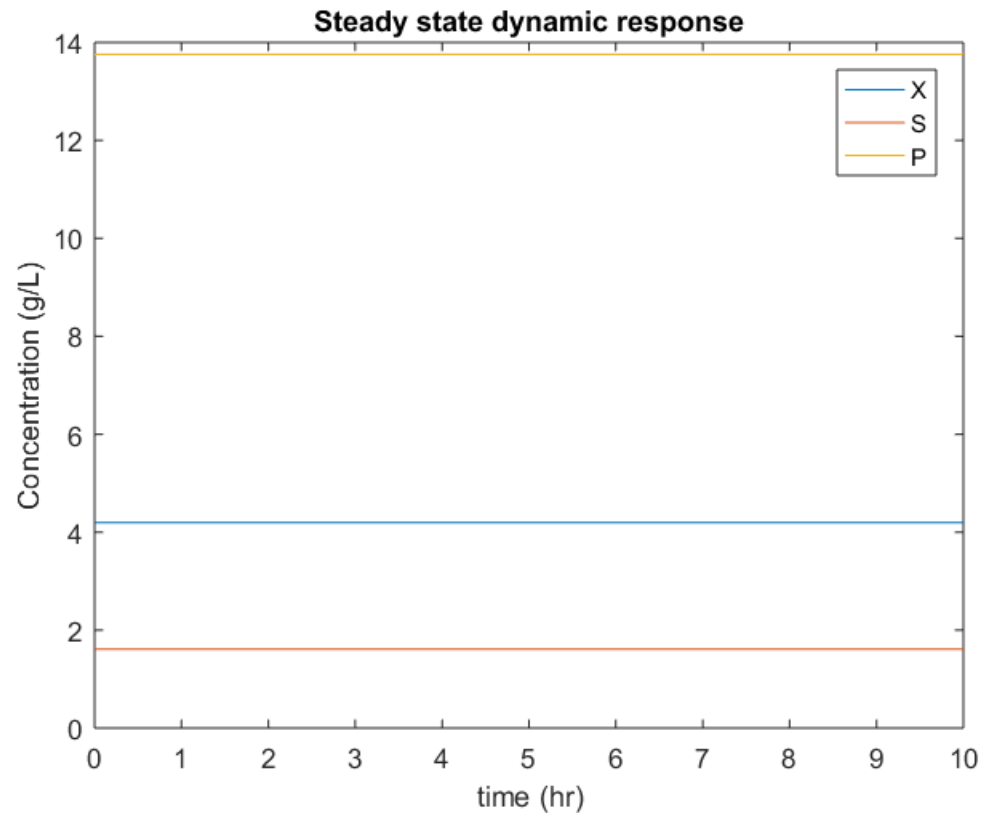
Question 3:
(See MATLAB code for implementation of ode45 function)

Question 4

```matlab
display('Question 4:')
display('(See figure 1)')
figure(1)
plot(t,sol)
legend('X','S','P'); xlabel('time (hr)'); ylabel('Concentration (g/
L)');
title('Steady state dynamic response');
display('The dynamic responses are straight lines because the system')
display('is initially at steady state. Differentials equal zero.')
display('---------------------------------------------')
```

Question 4:
(See figure 1)
The dynamic responses are straight lines because the system
is initially at steady state. Differentials equal zero.

Steady state dynamic response

Question 5

```
display('Question 5:')
D=0.20;        %/hr
p=[Y_XS alpha beta mu_m Pm Km Ki D Sf];
[t,sol]=ode45(@(t,x) bioreactor(x,p),tspan,F_SS2);
display('(See figure 2)')
figure(2)
plot(t,sol)
legend('X','S','P'); xlabel('time (hr)'); ylabel('Concentration (g/
L)');
title('Dynamic response with D=0.20 hr^{-1}');

D=0.30;        %/hr
p=[Y_XS alpha beta mu_m Pm Km Ki D Sf];
[t,sol]=ode45(@(t,x) bioreactor(x,p),tspan,F_SS2);
display('(See figure 3)')
figure(3)
plot(t,sol)
legend('X','S','P'); xlabel('time (hr)'); ylabel('Concentration (g/
L)');
title('Dynamic response with D=0.30 hr^{-1}');
set(gca,'FontSize',12); set(gca,'FontName','Times');
display('Biomass concentration increases with lower dilution rate,')
display('and decreases with higher dilution rate.')
```
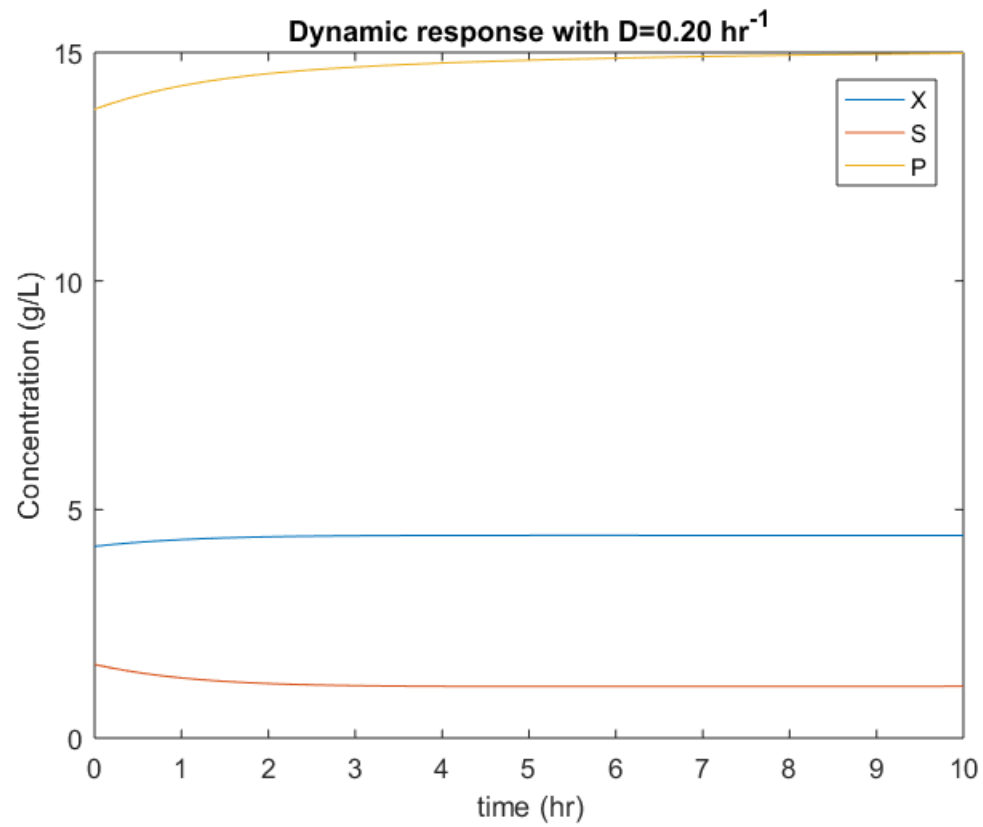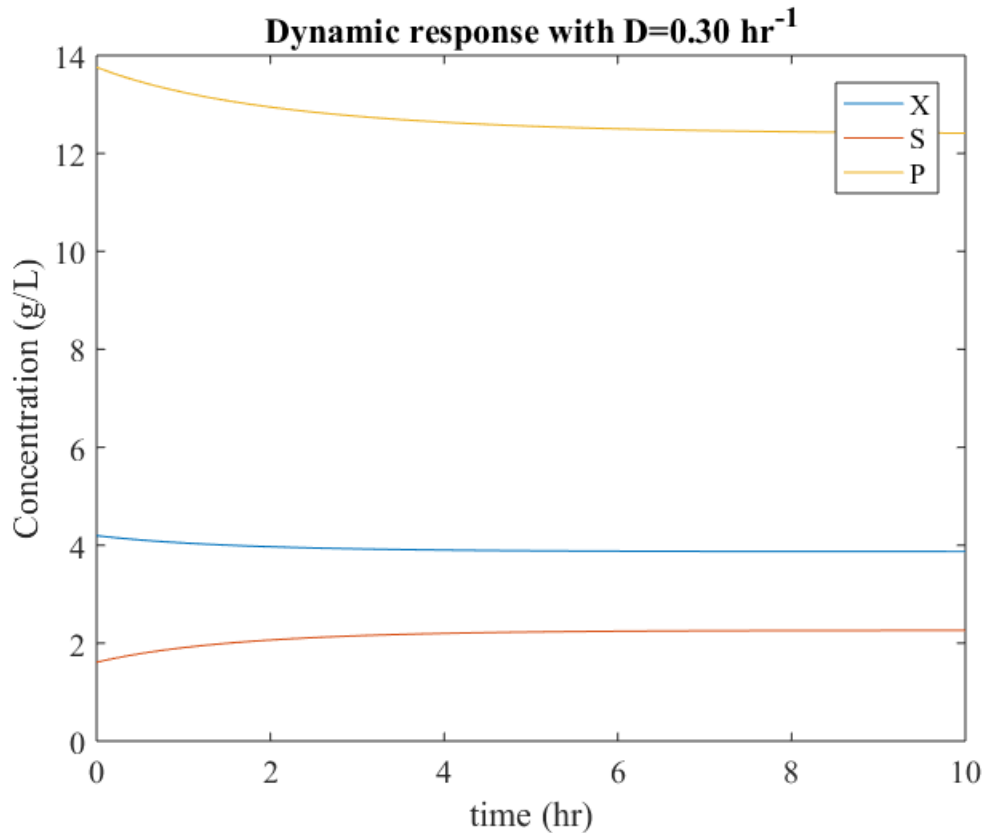
```
display('---------------------------------------------')
```

Question 5:
(See figure 2)
(See figure 3)
Biomass concentration increases with lower dilution rate,
and decreases with higher dilution rate.

Dynamic response with D=0.20 hr^{-1}

Dynamic response with D=0.30 hr^{-1}

Question 6

```
display('Question 6:')
% First find the new jacobian matrix using new diltion rate
D=0.20;      %/hr
p=[Y_XS alpha beta mu_m Pm Km Ki D Sf];
syms X S P
f=bioreactor([X S P],p);
A=jacobian(f,[X S P]);
X=F_SS2(1); S=F_SS2(2); P=F_SS2(3);
A=subs(A);
A=double(A);
% Solve system linearized using jacobian matrix
[t,sol]=ode45(@(t,x) bioreactor_linear(x,A),tspan,F_SS2);
display('(See figure 4)')
figure(4)
plot(t,sol)
legend('X','S','P'); xlabel('time (hr)'); ylabel('Concentration (g/
L)');
title('Linearized dynamic response with D=0.20 hr^{-1}');

% First find the new jacobian matrix using new diltion rate
D=0.30;      %/hr
p=[Y_XS alpha beta mu_m Pm Km Ki D Sf];
syms X S P
```

```matlab
f=bioreactor([X S P],p);
A=jacobian(f,[X S P]);
X=F_SS2(1); S=F_SS2(2); P=F_SS2(3);
A=subs(A);
A=double(A);
% Solve system linearized using jacobian matrix
[t,sol]=ode45(@(t,x) bioreactor_linear(x,A),tspan,F_SS2);
display('(See figure 5)')
figure(5)
plot(t,sol);
legend('X','S','P'); xlabel('time (hr)'); ylabel('Concentration (g/
L)');
title('Linear dynamic response with D=0.30 hr^{-1}');
display('Similar response in short time, however, non-realistic')
display('results with substrate concentration going negative.')
display('----------------------------------------------')

Question 6:
(See figure 4)
(See figure 5)
Similar response in short time, however, non-realistic
results with substrate concentration going negative.
----------------------------------------------
```
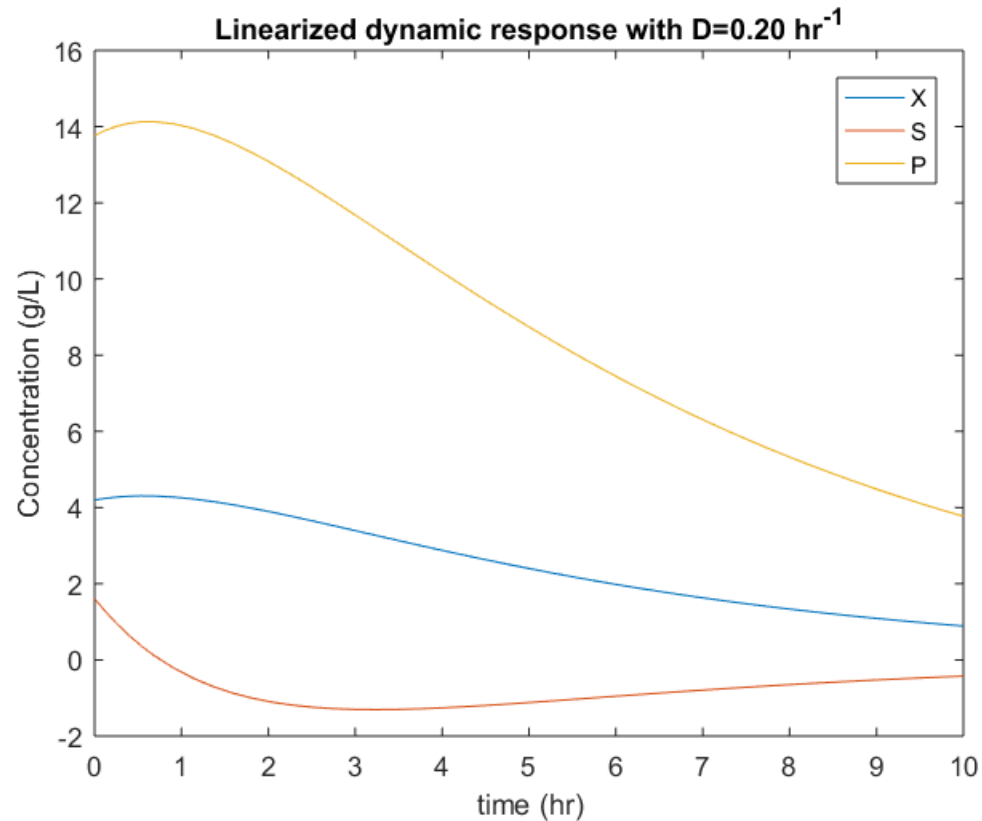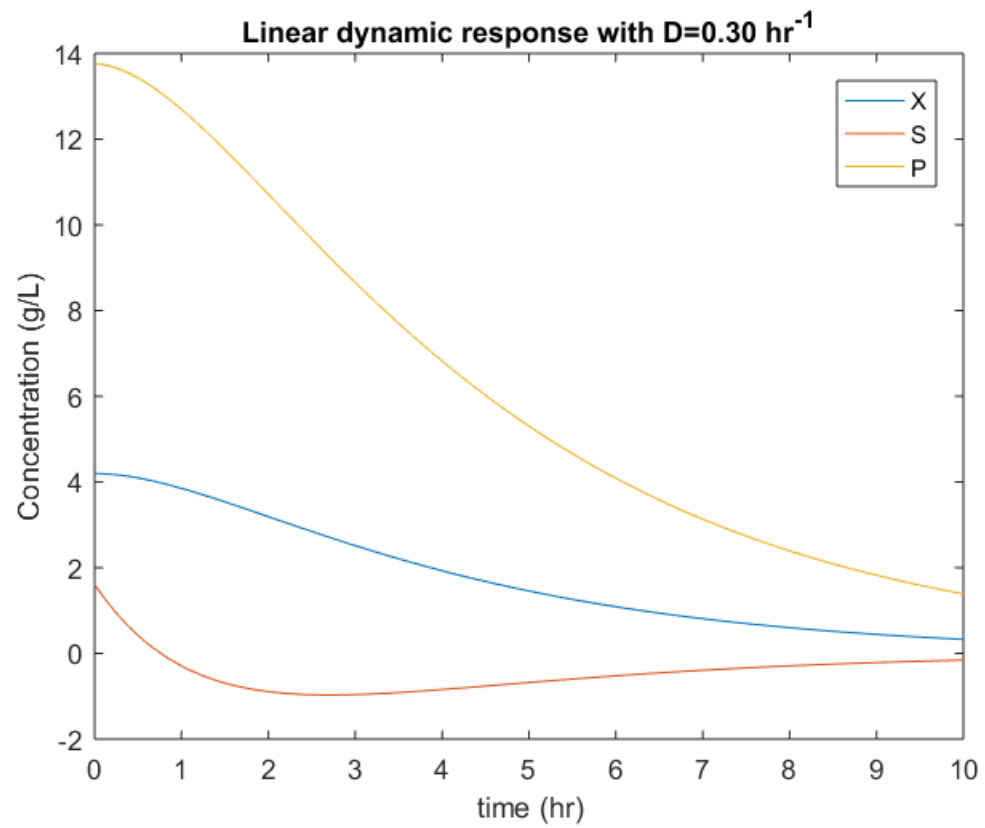

Linearized dynamic response with D=0.20 hr^{-1}

Linear dynamic response with D=0.30 hr^{-1}

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