MATLAB: Nonlinear Algebraic Systems

1. Functions for solving nonlinear equations
2. In-class exercise

MATLAB: Nonlinear Algebraic Systems

Functions for Solving Nonlinear Equations

Solving Single Equations: fzero

- fzero is a Matlab function that finds a root of a continuous function of one variable
- Syntax: x = fzero('fun',xo)
- 'fun' is the name of the user provided Matlab m-file function (fun.m) that evaluates and returns the LHS of $f(x) = 0$
- xo is an initial guess for the solution of $f(x) = 0$
- The algorithm uses a combination of bisection, secant and inverse quadratic interpolation methods

fzero Example

- Redlich-Kwong equation of state:

$$P = \frac{RT}{V-b} - \frac{a}{\sqrt{T}V(V+b)}$$

  - » P = pressure
  - » V = molar volume
  - » T = temperature
  - » R = gas constant
  - » a, b = gas dependent constants

- Given P, T, R, a and b, calculate V:

$$f(V) = P - \frac{RT}{V-b} + \frac{a}{\sqrt{T}V(V+b)} = 0$$

redlich.m

```
function f = redlich(x)

T = 390;

P = 174;

R = 0.08314;

a = 16.82;

b = 0.02219;

V = x;

 f = -P+R*T/(V-b)-a/(sqrt(T)*V*(V-b));
```

fzero Example

```
>>  vo=0.1;
>> v=fzero('redlich',vo)
v =
    0.1816
>> vo=1;
>> v=fzero('redlich',vo)
v =
    0.1816
>> vo=10;
>> v=fzero('redlich',vo)
Exiting fzero: aborting search for an interval containing a sign
change because no sign change is detected during search.
Function may not have a root.
v =
   NaN
```

Solving Equation Systems: fsolve

- fsolve is a MATLAB function for solving a system of nonlinear algebraic equations
- Syntax: x = fsolve('fun',xo)
  - » Same syntax as fzero, but x is a vector of variables and the function, 'fun', returns a vector of equation values, **f**(**x**)
- Multiple algorithms available in options settings (e.g. trust-region dogleg, Gauss-Newton, Levenberg-Marquardt)
- Part of the Optimization toolbox

fsolve_example.m

```
function f = fsolve_example(x)
f(1) = x(1)^2+x(2)^2-2;
f(2) = x(1)*x(2)-1;
>> xo = [10 10];
>> x=fsolve('fsolve_example',xo)
```

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} x_1^2 + x_2^2 - 2 \\ x_1 x_2 - 1 \end{bmatrix} = \mathbf{0}$$

Equation solved.

fsolve completed because the vector of function values is near zero as measured by the default value of the function tolerance, and the problem appears regular as measured by the gradient.

<stopping criteria details>

```
x =
    1.0000    1.0000
```

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