MATLAB: Linear ODE Systems

1. Introduction
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Introduction

Eigenvalue Calculation

$$\frac{d\mathbf{y}}{dt} = \begin{bmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\mathbf{y} \qquad \mathbf{y}(0) = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

```
>> A = [-6 -11 -6; 1 0 0; 0 1 0];
>> [X,D] = eig(A)
X =
   -0.9435    0.8729    0.5774
    0.3145   -0.4364   -0.5774
   -0.1048    0.2182    0.5774
D =
   -3.0000         0         0
         0   -2.0000         0
         0         0   -1.0000
```

```
>> X(:,1)
ans =
   -0.9435
    0.3145
   -0.1048
>> norm(X(:,1))
ans =
     1
```

Constant Determination

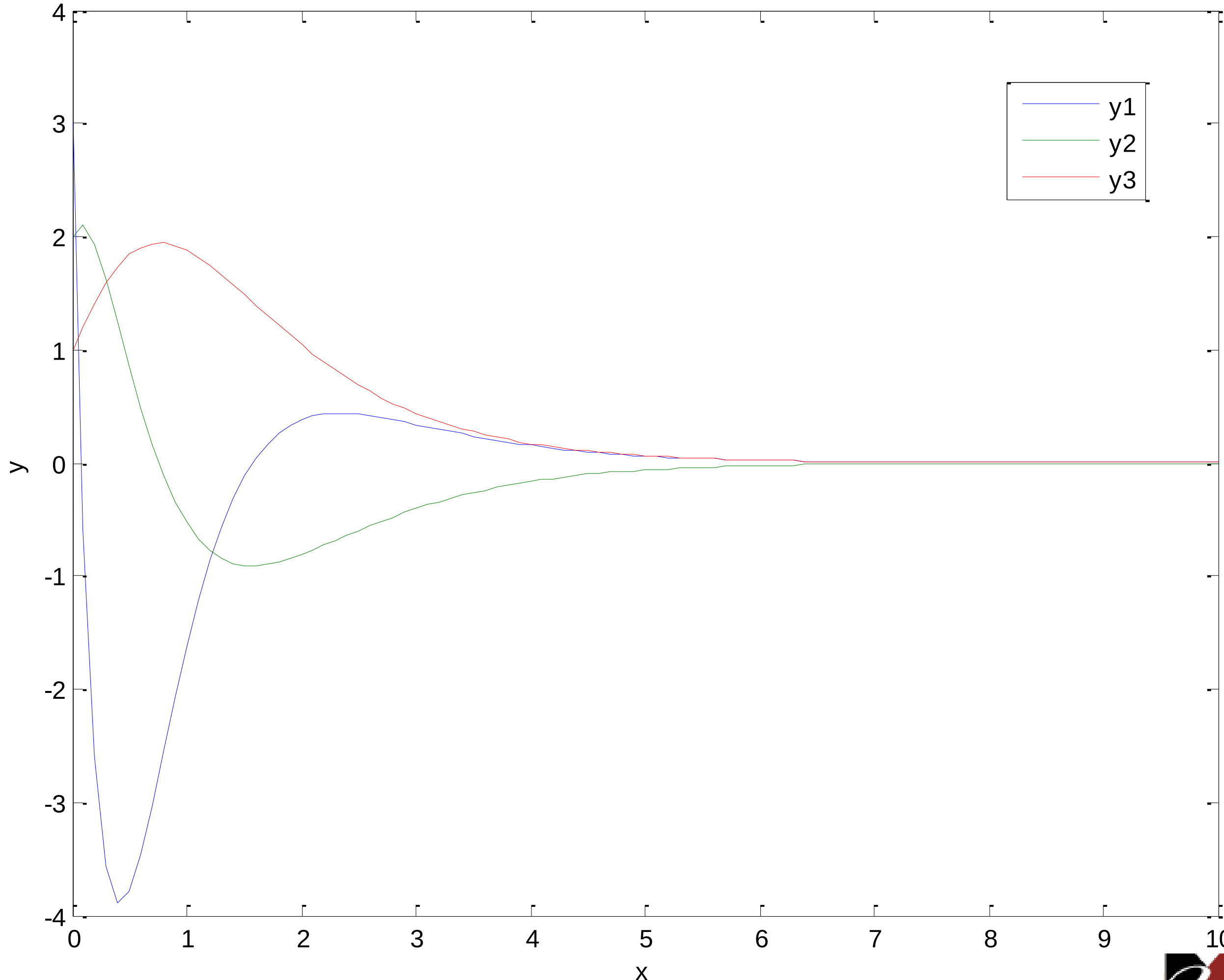
$$\mathbf{y}(x) = c_1 \mathbf{x}^{(1)} e^{\lambda_1 x} + c_2 \mathbf{x}^{(2)} e^{\lambda_2 x} + c_3 \mathbf{x}^{(3)} e^{\lambda_3 x}$$

$$\mathbf{c} = \mathbf{X}^{-1} \mathbf{y}(0)$$

```
>> y0 = [3 2 1]';
>> c = inv(X)*y0
c =
  -52.4667
  -64.1561
   16.4545
```

Plot Solution

```
>> t = [0:0.1:10];
>> y = c(1)*X(:,1)*exp(D(1,1).*t)+
c(2)*X(:,2)*exp(D(2,2).*t)+c(3)*X(:,3)*exp(D(3,3).*t);
>> plot(t,y)
>> ylabel('y')
>> xlabel('x')
>> legend('y1','y2','y3')
```



Linear State-Space Systems

- General form

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad \mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

  - » **x**(t) – state vector (dependent variables)
  - » *t* – independent variable (usually time)
  - » **u**(t) – input vector
  - » **y**(t) – output vector

- Restricted form

$$\frac{d\mathbf{x}}{dt} = \mathbf{A}\mathbf{x} \quad \mathbf{y} = \mathbf{I}\mathbf{x} = \mathbf{x} \quad \mathbf{B} = \mathbf{0}, \mathbf{C} = \mathbf{I}, \mathbf{D} = \mathbf{0}$$

Creating State-Space Models

sys = ss(A,B,C,D) creates an object sys representing the state-space model

```
>> A = [-6 -11 -6; 1 0 0; 0 1 0];
>> B = [];
>> C = eye(3);
>> D = [];
```

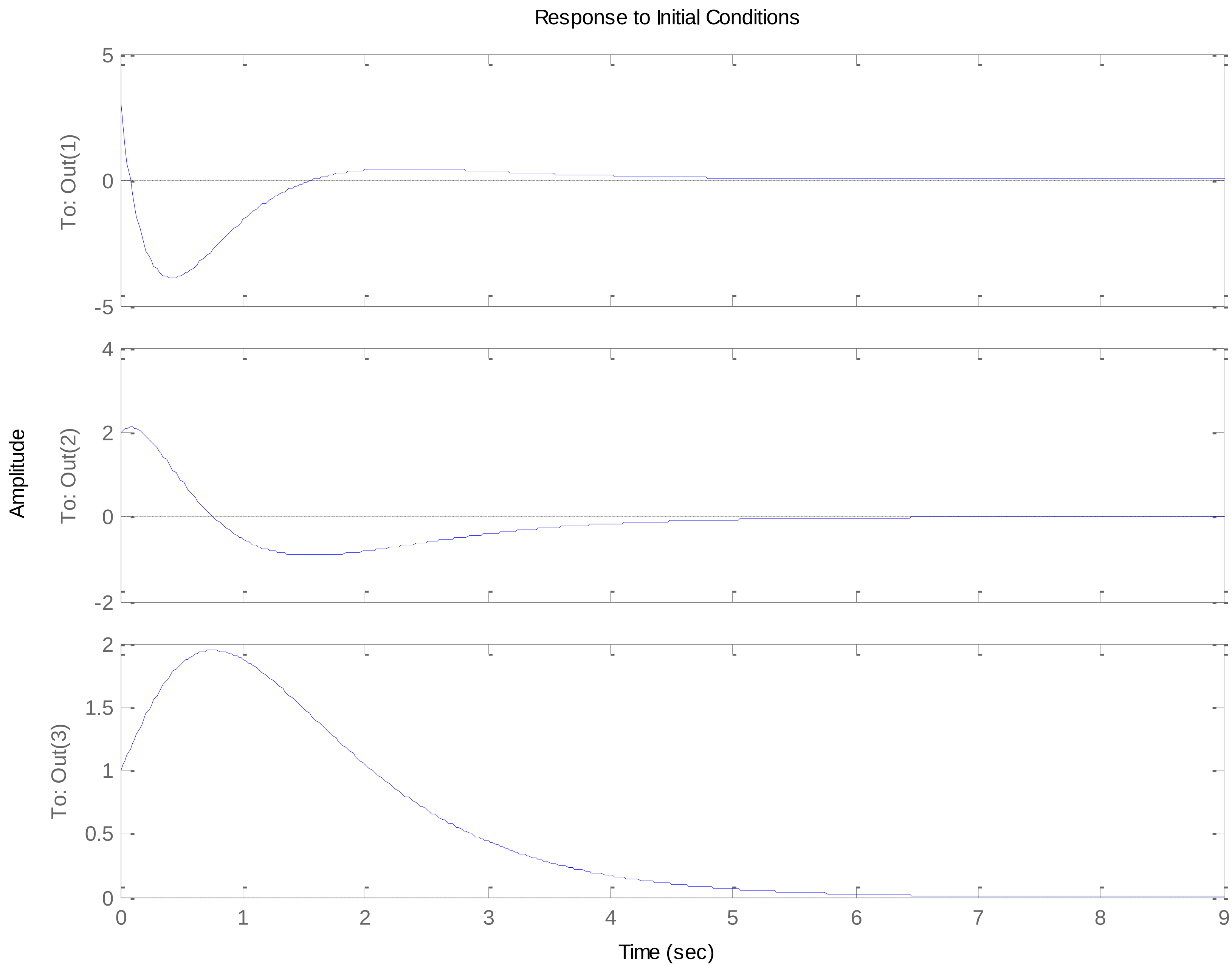
```
>> sys = ss(A,B,C,D)
a =
        x1   x2   x3
   x1   -6  -11   -6
   x2    1    0    0
   x3    0    1    0
b =
     Empty matrix: 3-by-0
c =
        x1  x2  x3
   y1   1   0   0
   y2   0   1   0
   y3   0   0   1
d =
     Empty matrix: 3-by-0
```

Generate and Plot Solutions

- initialplot(sys,x0)
  - » Plots the undriven response of the state-space model sys (created with ss) with initial condition x0 on the states
  - » The response corresponds to the equations: dx/dt = Ax , y = Cx , x(0) = x0

```
>> x0 = [3 2 1]';
>> initialplot(sys,x0)
Warning: Some of the specified systems have no input and/or output.
```

Generate and Plot Solutions



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In-class Exercise

Batch Chemical Reactor

- Reaction sequence

$$A \underset{k_2}{\overset{k_1}{\rightleftarrows}} B, \qquad B \xrightarrow{k_3} C \xrightarrow{k_5} E, \qquad B \xrightarrow{k_4} D$$

- Batch chemical reactor model

$$\frac{dC_A}{dt} = -k_1 C_A + k_2 C_B \qquad C_A(0) = 10$$

$$\frac{dC_B}{dt} = k_1 C_A - k_2 C_B - k_3 C_B - k_4 C_B \qquad C_B(0) = 0$$

$$\frac{dC_C}{dt} = k_3 C_B - k_5 C_c \qquad C_C(0) = 0$$