Matlab Homework #3
ChE 231
Spring 2019

Consider a biochemical reactor where biomass and a desired product are produced by microbial growth from a single, rate limiting substrate. Denote the biomass concentration as X, the substrate concentration as S, and the product concentration as P. The substrate is continuously supplied to the bioreactor at volumetric flow rate F and concentration S_f. A liquid stream is continuously removed from the bioreactor at volumetric flow rate F such that the liquid volume V remains constant. The specific growth rate has the form:

$$\mu(S,P) = \frac{\mu_m\left(1-\frac{P}{P_m}\right)S}{K_m + S + \frac{S^2}{K_i}}$$

where the constants K_i and P_m reflect growth inhibition by high concentrations of substrate and product, respectively. The bioreactor is described by the following mass balance equations:

$$\frac{dX}{dt} = -DX + \mu(S,P)X = f_1(X,S,P)$$

$$\frac{dS}{dt} = D\left(S_f - S\right) - \frac{1}{Y_{X/S}}\mu(S,P)X = f_2(X,S,P)$$

$$\frac{dP}{dt} = -DP + (\alpha\mu(S,P) + \beta)X = f_3(X,S,P)$$

where $D = F/V$. Consider the following parameter values: $Y_{X/S}$ = 0.5 g/g, α = 2.8 g/g, β = 0.12 hr^{-1}, μm = 0.75 hr^{-1}, P_m = 60 g/L, K_m = 2.0 g/L, K_i = 23 g/L, D = 0.25 hr^{-1}, and S_f = 10 g/L.

1. Develop a M-file to solve the steady-state model equations with the Matlab code fsolve. Find the washout steady state and the non-trivial steady state.
2. Linearize the nonlinear differential equation model about the non-trivial steady state to obtain the linearized model $\frac{dX}{dt} = \boldsymbol{A}x$. Determine if the steady state is stable by using the Matlab function eig to calculate the eigenvalues of the **A** matrix.
3. Use the M-file in Part 1 to solve the nonlinear differential equation model using one of the Matlab integration codes. Make the program sufficiently general such that different integration codes can be used by simply changing the function call.
4. Test your Matlab code by integrating the nonlinear differential equation model with the initial condition set equal to the steady state in Part 1. Plot the dynamics responses X(t), S(t), and P(t). What do you expect these dynamic responses to look like?

5. Integrate the nonlinear differential equation model with the same initial condition (as Part 4) for the dilution rate D = 0.20 hr^{-1}. Repeat this test for D = 0.30 hr^{-1}. Plot the dynamics responses X(t), S(t), and P(t) for these two dilution rates. What happens to the biomass concentration for these two cases?
6. Develop a M-file to solve the linearized model in Part 2 using the same Matlab integration code as used in Part 3. Integrate the linearized model with the same initial condition (as Parts 4 & 5) for the dilution rates D = 0.20 hr^{-1} and D = 0.30 hr^{-1}. Plot the dynamics responses X(t), S(t), and P(t). Compare the linear model responses to the nonlinear model responses in Part 5.