**Written HW 2 Solutions**

24.5, Problem 6

$P(X = 0) = \frac{4}{10}.\frac{3}{9} = 0.133 = 13\frac{1}{3}\%$

$P(X = 1) = \frac{6}{10}.\frac{4}{9} + \frac{4}{10}.\frac{6}{9} = 0.533 = 53\frac{1}{3}\%$

$P(X = 2) = \frac{6}{10}.\frac{5}{9} = 0.33 = 33\frac{1}{3}\%$

$P(1 < X < 2) = 0$ since X is a whole number between 0 and 6

$P(X \leq 1) = P(X = 0) + P(X = 1) = 0.667 = 66\frac{2}{3}\%$

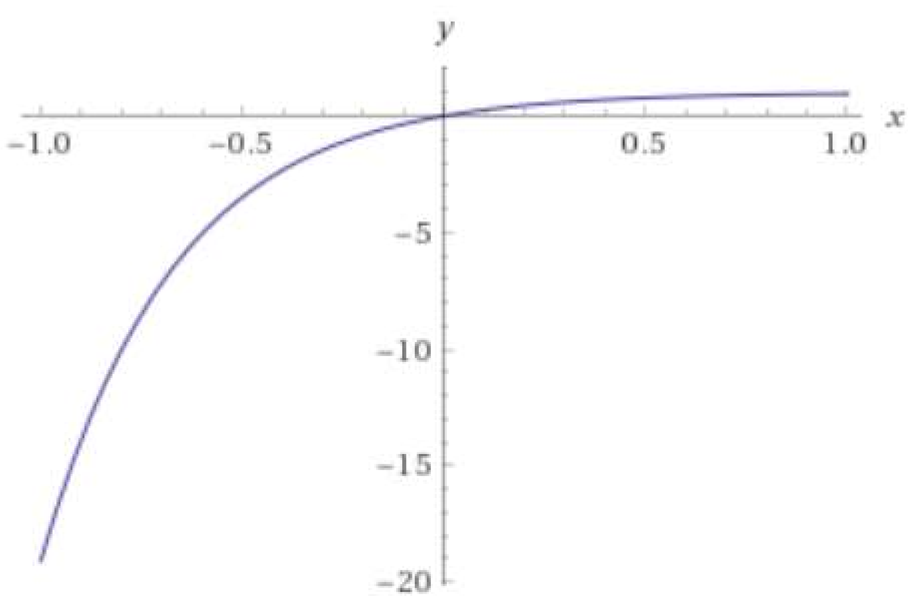
$P(X \geq 1) = P(X = 1) + P(X = 2) = 0.867 = 86\frac{2}{3}\%$

$P(X > 1) = 1 - P(X \leq 1) = 1 - 0.667 = 0.33 = 33\frac{1}{3}\%$

$P(0.5 < X < 10) = 1 - P(X = 0) = 1 - 0.133 = 86\frac{2}{3}\%$

24.5, Problem 8

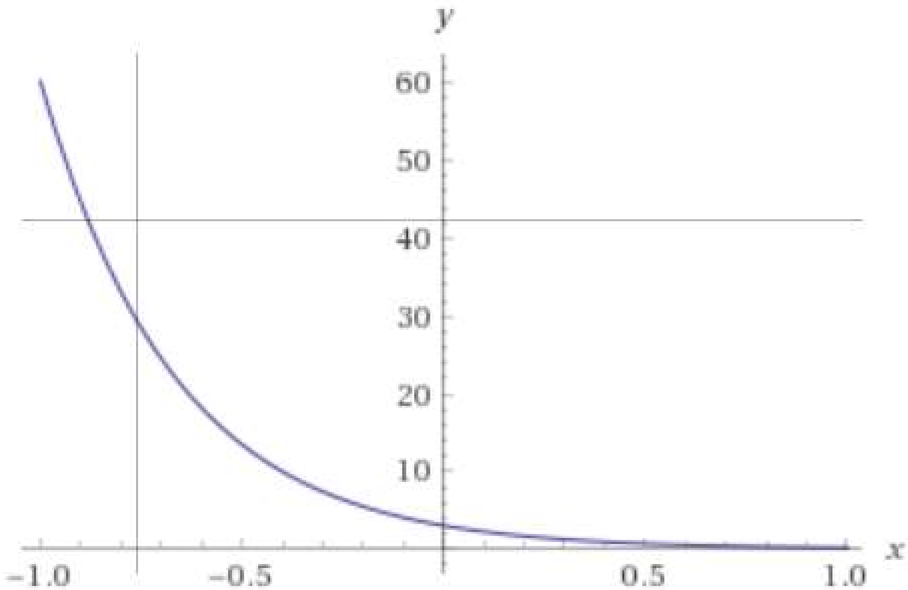
Plot of F(x)




$f(x) = F'(x)$ gives $f(x) = 0$ if $x<0$, $f(x) = 3e^{-3x}$ if $x>0$

If $F(x) = 0.9$, then $0.1 = e^{-3x}, x = \frac{1}{3}ln10 = 0.7675$

Plot of $f(x)$




24.6, Problem 12

For rolling a fair die 20 times. Possible outcomes are 1,2,3,4, and 6 each with a probability of $\frac{1}{6}$

So, the sum can be $20.\frac{1}{6}.(1+2+3+4+5+6) = 70$ or a number around it.

24.6, Problem 18

$f(x) = 0.001e^{-0.001x}$ $(x \geqq 0)$

Writing $k = 0.001$ (and in fact for any positive $k$) we obtain, by integrating by parts,

$$\mu = k\int_0^\infty xe^{-kx}\,dx = \frac{1}{k}.$$

24.7, Problem 12

Using Poisson Distribution,

$$f(x) = \frac{\mu^x}{x!}e^{-\mu}$$

**(a)** $\mu = 6$ defects per 300 m, $f(x) = 6^x e^{-6}/x!$,
**(b)** $f(0) = e^{-6} = 0.00248 \approx 0.25\%$

24.7, Problem 14

Let $X$ be the number of customers per minute. The average number is $120/60 = 2$ per minute. Hence $X$ has a Poisson distribution with mean 2. Waiting occurs if $X > 4$. The probability of the complement is $P(X \leqq 4) = 0.9473$ (see Table 6). Hence the answer is $1 - 0.9473 = 5\frac{1}{4}\%$.

24.8, Problem 8

We get the maximum load $c$ from the condition

$$P(X \leqq c) = \Phi\left(\frac{c - 1500}{50}\right) = 5\%.$$

By Table A8 in Appendix 5.

$$\frac{c - 1500}{50} = -1.645. \qquad c = 1418 \text{ kg}.$$

24.8, Problem 10

Applying the De Moivre–Laplace theorem. We get

$$\begin{aligned} P &= \sum_{x=0}^{10} \binom{1000}{x} 0.01^x 0.99^{1000-x} \\ &\approx \Phi\left(\frac{10 - 10 + 0.5}{\sqrt{9.9}}\right) - \Phi\left(\frac{0 - 10 - 0.5}{\sqrt{9.9}}\right) = 0.564. \end{aligned}$$

The value of the binomial distribution in the first line is 0.583; the relative error of the approximation is about $3\frac{1}{4}\%$.