11

HEAT TRANSFER AND ENERGY CONSERVATION

Example

A VARIETY OF AVAILABLE energy conservation measures can be adopted to optimize energy usage throughout a chemical plant or refinery. The following is a representative list of design or operating factors related to heat transfer and energy use that can involve optimization:

1. Fired heater combustion controls
2. Heat recovery from stack gases
3. Fired heater convection section cleaning
4. Heat exchanger network configuration
5. Extended surface heat exchanger tubing to improve heat transfer
6. Scheduling of heat exchanger cleaning
7. Air cooler performance
8. Fractionating towers: optimal reflux ratio, heat exchange, and so forth
9. Instrumentation for monitoring energy usage
10. Reduced leakage in vacuum systems and pressure lines and condensers
11. Cooling water savings
12. Efficient water treatment for steam raising plants
13. Useful work from steam pressure reduction
14. Steam traps, tracing, and condensate recovery
15. CO boilers on catalytic cracking units
16. Electrical load leveling
17. Power factor improvement
18. Power recovery from gases or liquids
19. Loss control in refineries
20. Catalyst improvements

Many of the conservation measures require detailed process analysis plus optimization. For example, the efficient firing of fuel (category 1) is extremely important in all applications. For any rate of fuel combustion, a theoretical quantity of air (for complete combustion to carbon dioxide and water vapor) exists under which the most efficient combustion occurs. Reduction of the amount of air available leads to incomplete combustion and a rapid decrease in efficiency. In addition, carbon particles may be formed that can lead to accelerated fouling of heater tube surfaces. To allow for small variations in fuel composition and flow rate and in the air flow rates that inevitably occur in industrial practice, it is usually desirable to aim for operation with a small amount of excess air, say 5 to 10 percent, above the theoretical amount for complete combustion. Too much excess air, however, leads to increased sensible heat losses through the stack gas.

In practice, the efficiency of a fired heater is controlled by monitoring the oxygen concentration in the combustion products in addition to the stack gas temperature. Dampers are used to manipulate the air supply. By tying the measuring instruments into a feedback loop with the mechanical equipment, optimization of operations can take place in real time to account for variations in the fuel flow rate or heating value.

As a second example (category 4), a typical plant contains large numbers of heat exchangers used to transfer heat from one process stream to another. It is important to continue to use the heat in the streams efficiently throughout the process. Incoming crude oil is heated against various product and reflux streams before entering a fired heater in order to be brought to the desired fractionating column flash zone temperature. Among the factors that must be considered in design or retrofit are

1. What should be the configuration of flows (the order of heat exchange for the crude oil)?
2. How much heat exchange surface should be supplied within the chosen configuration?

Additional heat exchange surface area leads to improved heat recovery in the crude oil unit but increases capital costs so that increasing the heat transfer surface area soon reaches diminishing returns. The optimal configuration and areas selected, of course, are strongly dependent on fuel costs. As fuel costs rise, existing plants can usually profit from the installation of additional heat exchanger surface in circumstances previously considered only marginally economic.

As a final example (category 6), although heat exchangers may be very effective when first installed, many such systems become dirty in use and heat transfer rates deteriorate significantly. It is therefore often useful to establish optimal heat exchanger cleaning schedules. Although the schedules can be based on observations of the actual deterioration of the overall heat transfer of the exchanger in question, it is also possible to optimize the details of the cleaning schedules depending on an economic assessment of each exchanger.

In this chapter we illustrate the application of various optimization techniques to heat-transfer-system design. First we show how simple rules of thumb on boiler temperature differences can be derived (Example 11.1). Then a more complicated design of a heat exchanger is examined (Example 11.2), leading to a constrained optimization problem involving some discrete-valued variables. Example 11.3 discusses the use of optimization in the design and operation of evaporators, and we conclude this chapter by demonstrating how linear programming can be employed to optimize a steam/power system (Example 11.4). For optimization of heat exchanger networks by mathematical programming methods, refer to Athier et al. (1997), Briones and Kokossis (1996), and Zamora and Grossmann (1998).

EXAMPLE 11.1 OPTIMIZING RECOVERY OF WASTE HEAT

A variety of sources of heat at elevated temperatures exist in a typical chemical plant that may be economically recoverable for production of power using steam or other working fluids, such as freon or light hydrocarbons. Figure E11.1 is a schematic of such a system. The system power output can be increased by using larger heat exchanger surface areas for both the boiler and the condenser. However, there is a trade-off between power recovery and capital cost of the exchangers. Jegede and Polley (1992), Reppich and Zagermann (1995), Sama (1983), Swearingen and Ferguson (1984), and Steinmeyer (1984) have proposed some simple rules based on analytical optimization of the boiler ΔT.

In a power system, the availability expended by any exchanger is equal to the net work that could have been accomplished by having each stream exchange heat with the surroundings through a reversible heat engine or heat pump. In the boiler in Figure E11.1, heat is transferred at a rate Q (the boiler load) from the average hot fluid

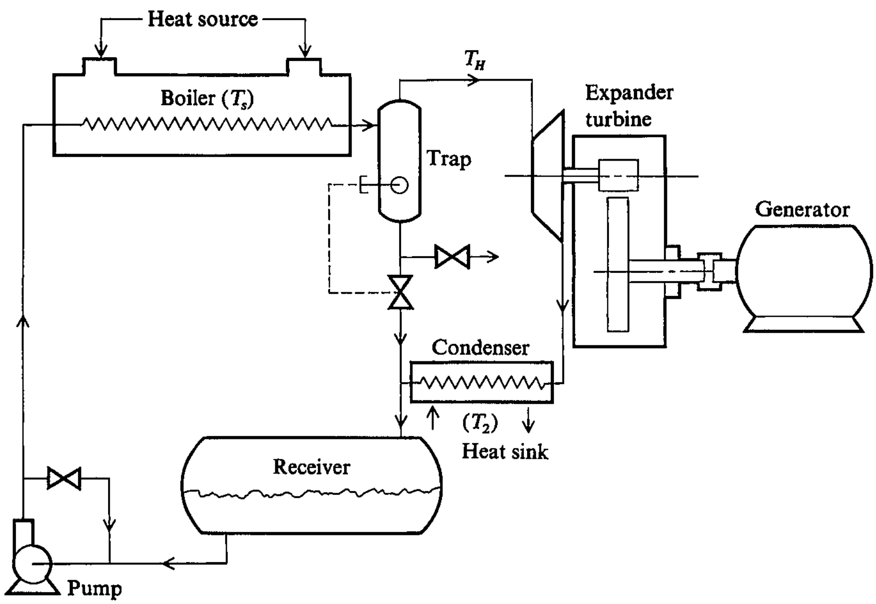

FIGURE E11.1
Schematic of power system.

temperature T_s to the working fluid at T_H. The working fluid then exchanges heat with the condenser at temperature T_2. If we ignore mechanical friction and heat leaks, the reversible work available from Q at temperature T_s with the condensing (cold-side) temperature at T_2 is

$$W_1 = Q\left(\frac{T_s - T_2}{T_s}\right) \qquad (a)$$

The reversible work available from the condenser using the working fluid temperature T_H (average value) and the heat sink temperature T_2 is

$$W_2 = Q\left(\frac{T_H - T_2}{T_H}\right) \qquad (b)$$

Hence the ideal power available from the boiler can be found by subtracting W_2 from W_1

$$W_2 - W_1 = \Delta W = Q\left(\frac{T_2}{T_H} - \frac{T_2}{T_s}\right) \qquad (c)$$

In this expression T_s and T_2 are normally specified, and T_H is the variable to be adjusted. If Q is expressed in Btu/h, and the operating cost is C_{op}, then the value of the available power is

$$C_{\text{op}} = C_H \eta y Q\left(\frac{T_2}{T_H} - \frac{T_2}{T_s}\right) \qquad (d)$$

where η = overall system efficiency (0.7 is typical)
y = number of hours per year of operation
C_H amalgamates the value of the power in \$/kWh and the necessary conversion factors to have a consistent set of units

You can see, using Equation (*d*) only, that C_{op} is minimized by setting $T_H = T_s$ (infinitesimal boiler ΔT). However, this outcome increases the required boiler heat transfer area to an infinite area, as can be noted from the calculation for the area

$$A = \frac{Q}{U(T_s - T_H)} \tag{e}$$

(In Equation (*e*) an average value for the heat transfer coefficient U is assumed, ignoring the effect of pressure drop. U depends on the working fluid and the operating temperature.) Let the cost per unit area of the exchanger be C_A and the annualization factor for capital investment be denoted by r. Then the annualized capital cost for the boiler is

$$C_c = \frac{C_A Q r}{U(T_s - T_H)} \tag{f}$$

Finally, the objective function to be minimized with respect to T_H, the working fluid temperature, is the sum of the operating cost and surface area costs:

$$f = C_H \eta y Q\left(\frac{T_2}{T_H} - \frac{T_2}{T_s}\right) + \frac{C_A Q r}{U(T_s - T_H)} \tag{g}$$

To get an expression for the minimum of f, we differentiate Equation (*g*) with respect to T_H and equate the derivative to zero to obtain

$$C_H \eta y Q\left(-\frac{T_2}{T_H^2}\right) + \frac{C_A Q r}{U(T_s - T_H)^2} = 0 \tag{h}$$

To solve the quadratic equation for T_H, let

$$\alpha_1 = C_H \eta y T_2 U$$

$$\alpha_2 = C_A r$$

Q cancels in both terms. On rearrangement, the resulting quadratic equation is

$$(\alpha_1 - \alpha_2)T_H^2 - 2\alpha_1 T_s T_H + \alpha_1 T_s^2 = 0 \tag{i}$$

The solution to (*i*) for $T_H < T_s$ is

$$T_H = T_s\left(\frac{\alpha_1 - \sqrt{\alpha_1 \alpha_2}}{\alpha_1 - \alpha_2}\right) \tag{j}$$

For a system with $C_A = \$25/\text{ft}^2$, a power cost of \$0.06/kWh ($C_H = 1.76 \times 10^{-5}$). $U = 95$ Btu/(h)(°R)(ft^2), $y = 8760$ h/year, $r = 0.365$, $\eta = 0.7$, $T_2 = 600°R$, and $T_s = 790°R$, the optimal value T_H is 760.7°R, giving a ΔT of 29.3°R. Swearingen and Ferguson showed that Equation (h) can be expressed implicitly as

$$\Delta T = T_s - T_H = T_H\left(\frac{\alpha_1}{\alpha_2}\right)^{1/2} \tag{k}$$

In this form, it appears that the allowable ΔT increases as the working fluid temperature increases. This suggests that the optimum ΔT for a heat source at 900°R is lower than that for a heat source at 1100°R. In fact, Equation (j) indicates that the optimum ΔT is directly proportional to T_s. Sama argues that this is somewhat counterintuitive because the Carnot "value" of a high-temperature source implies using a smaller ΔT to reduce lost work.

The working fluid must be selected based on the heat source temperature, as discussed by Swearingen and Ferguson. See Sama for a discussion of optimal temperature differences for refrigeration systems; use of Equation (k) leads to ΔT's ranging from 8 to 10°R.

EXAMPLE 11.2 OPTIMAL SHELL-AND-TUBE HEAT EXCHANGER DESIGN

In this example we examine a procedure for optimizing the process design of a baffled shell-and-tube, single-pass, counterflow heat exchanger (see Figure E11.2a), in which the tube fluid is in turbulent flow but no change of phase of fluids takes place in the shell or tubes. Usually the following variables are specified a priori by the designer:

1. Process fluid rate (the hot fluid passes through the tubes), W_i
2. Process fluid temperature change, $T_2 - T_1$

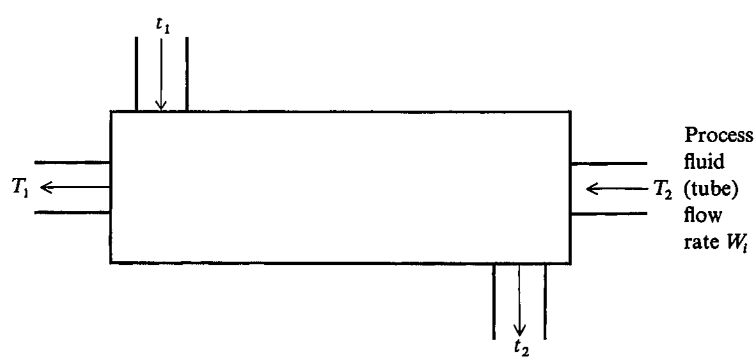

FIGURE E11.2a
Process diagram of shell-and-tube counterflow heat exchanger. *Key:* $\Delta t_l = T_l - t_l$ cold-end temperature difference; $\Delta t_2 = T_2 - t_2$ warm-end temperature difference.

3. Coolant inlet temperature (the coolant flows through the shell), t_1
4. Tube spacing and tube inside and outside diameters (D_i, D_o).

Conditions 1 and 2 imply the heat duty Q of the exchanger is known.

The variables that might be calculated via optimization include

1. Total heat transfer area, A_o
2. Warm-end temperature approach, Δt_2
3. Number and length of tubes, N_t and L
4. Number of baffle spacings, n_b
5. Tube-side and shell-side pressure drop
6. Coolant flow, W_c

Not all of these variables are independent, as shown in the following discussion.

In contrast to the analysis outlined in Example 11.1, the objective function in this example does not make use of reversible work. Rather, a cost is assigned to the usage of coolant as well as to power losses because of the pressure drops of each fluid. In addition, annualized capital cost terms are included. The objective function in dollars per year is formulated using the notation in Table E11.2A

$$C = C_c W_c y + C_A A_o + C_i E_i A_o + C_o E_o A_o \tag{a}$$

Suppose we minimize the objective function using the following set of four variables, a set slightly different from the preceding list.

1. Δt_2: warm-end temperature difference
2. A_o: tube outside area
3. h_i: tube inside heat transfer coefficient
4. h_o: tube outside heat transfer coefficient

Only three of the four variables are independent. If A_o, h_i, and h_o are known, then Δt_2 can be found from the heat duty of the exchanger Q:

$$Q = F_t U_o A_o \frac{\Delta t_2 - \Delta t_1}{\ln(\Delta t_2 / \Delta t_1)} \tag{b}$$

F_t is unity for a single-pass exchanger. U_o is given by the values of h_o, h_i, and the fouling coefficient h_t as follows:

$$\frac{1}{U_o} = \frac{1}{f_A h_i} + \frac{1}{h_o} + \frac{1}{h_t} \tag{c}$$

Cichelli and Brinn (1956) showed that the annual pumping loss terms in Equation (a) could be related to h_i and h_o by using friction factor and j-factor relationships for tube flow and shell flow:

$$E_i = \phi_i h_i^{3.5} \tag{d}$$

$$E_0 = \phi_0 h_0^{4.75} \tag{e}$$

TABLE E11.2A

Nomenclature for heat exchanger optimization

A_{lm}	Log mean of inside and outside tube surface areas
A_i	Inside tube surface area, ft^2
A_o	Outside tube surface area, ft^2
C	Total annual cost, \$/year
C_A	Annual cost of heat exchanger per unit outside tube surface area, \$/(ft^2)(year)
C_c	Cost of coolant, \$/lb mass
C_i	Annual cost of supplying 1(ft)(lb$_f$)/h to pump fluid flowing inside tubes, (\$)(h)/(ft)(lb$_f$)(year)
C_o	Annual cost of supplying 1(ft)(lb$_f$)/h to pump shell side fluid, (\$)(h)/(ft)(lb$_f$)(year)
c	Specific heat at constant pressure, Btu/(lb$_m$)(°F)
D_i	Tube inside diameter, ft
D_o	Tube outside diameter, ft
E_i	Power loss inside tubes per unit outside tube area, (ft)(lb$_f$)/(ft^2)(h)
E_o	Power loss outside tubes per unit outside tube area, (ft)(lb$_f$)/(ft^2)(h)
f	Friction factor, dimensionless
f_A	A_i/A_o
F_t	Multipass exchanger factor
g_c	Conversion factor, (ft)(lb$_m$)/(lb$_f$)(h^2) = 4.18×10^8
h_f	Fouling coefficient
h_i	Coefficient of heat transfer inside tubes, Btu/(h)(ft^2)(°F)
h_o	Coefficient of heat transfer outside tubes, Btu/(h)(ft^2)(°F)
h_t	Combined coefficient for tube wall and dirt films, based on tube outside area Btu/(h)(ft^2)(°F) $\frac{1}{h_t} = \frac{L'A_o}{k_w A_{1m}} + \frac{1}{h_{f_i}}\frac{A_o}{A_i} + \frac{1}{h_{f_o}}$
k	Thermal conductivity, Btu/(h)(ft)(°F)
L	Lagrangian function
L_t	Length of tubes, ft
L'	Thickness of tube wall, ft

(continued)

The coefficients ϕ_i and ϕ_o depend on fluid specific heat c, thermal conductivity k, density ρ, and viscosity μ, as well as the tube diameters. ϕ_o is based on either in-line or staggered tube arrangements.

If we solve for W_c from the energy balance

$$W_c = \frac{Q}{c(\Delta t_1 - \Delta t_2 + T_2 - T_1)} \tag{f}$$

and substitute for E_i, E_o, and W_c in Equation (*a*), the resulting objective function is

$$f = \frac{C_c y Q}{c(\Delta t_1 - \Delta t_2 + T_2 - T_1)} + C_A A_o + C_i \phi_i h_i^{3.5} A_o + C_o \phi_o h_o^{4.75} A_o \tag{g}$$

TABLE E11.2A (CONTINUED)
Nomenclature for heat exchanger optimization

n_b	Number of baffle spacing on shell side = number of baffles plus 1
N_c	Number of clearances for flow between tubes across shell axis
N_t	Number of tubes in exchanger
Δp_i	Pressure drop for flow through tube side, lb_f/ft^2
Δp_o	Pressure drop for flow through tube side, lb_f/ft^2
Q	Heat transfer rate in heat exchanger, Btu/h
S_0	Minimum cross-sectional area for flow across tubes, ft^2
T_1	Outlet temperature of process fluid, °F
T_2	Inlet temperature of process fluid, °F
t_1	Inlet temperature of coolant, °F
t_2	Outlet temperature of coolant, °F
ΔT_1	$T_1 - t_1$, = cold-end temperature difference
ΔT_2	$T_2 - t_2$, = warm-end temperature difference
U_o	Overall coefficient of heat transfer, based on outside tube area, Btu/(h)(ft^2)(°F)
v_i	Average velocity of fluid inside tubes, ft/h
v_o	Average velocity of fluid outside tubes, ft/h at shell axis
W_c	Coolant rate, lb/h
W_i	Flow rate of fluid inside tubes, lb_m/h
W_o	Flow rate of fluid outside tubes, lb_m/h
y	Operating hours per year
ρ_i	Density of fluid inside tubes, lb_m/ft^3
ρ_o	Density of fluid outside tubes, lb_m/ft^3
μ	Viscosity of fluid, lb_m/(h)(ft)
ϕ_i	Factor relating friction loss to h_i
ϕ_o	Factor relating friction loss to h_o
ω	Lagrange multiplier
Subscripts	
c	Coolant
f	Film temperature, midway between bulk fluid and wall temperature
i	Inside the tubes
o	Outside the tubes
w	Wall

To accommodate the constraint (b), a Lagrangian function L is formed by augmenting f with Equation (b), using a Lagrange multiplier ω

$$L = f + \omega\left[\frac{F_t(\Delta t_2 - \Delta t_1)}{Q \ln(\Delta t_2/\Delta t_1)} - \frac{1}{U_o A_o}\right] \qquad (h)$$

Equation (h) can be differentiated with respect to four variables (h_i, h_o, Δt_2, and A_o). After some rearrangement, you can obtain a relationship between the optimum h_o and h_i, namely

$$h_o = \left(\frac{0.74 C_i \phi_i f_A}{C_o \phi_o}\right)^{0.17} h_i^{0.78} \qquad (i)$$

This is the same result as derived by McAdams (1942), having the interpretation that the friction losses in the shell and tube sides, and the heat transfer resistances must be balanced economically. The value of h_i can be obtained by solving

$$C_A - 2.5C_i\phi_i h_i^{3.5} - 2.91(C_o\phi_o)^{0.17}(C_i\phi_i f_A)^{0.83}h_i^{3.72} - \frac{3.5C_i\phi_i f_A h_i^{4.5}}{h_t} = 0 \quad (j)$$

The simultaneous solution of Equations (f), (i), and (j) yields another expression:

$$\frac{C_c y U_o}{c(C_A + C_i E_i + C_o E_o)} = \left(1 + \frac{T_2 - T_1}{\Delta t_2 - \Delta t_1}\right)^2\left[\ln\left(\frac{\Delta t_2}{\Delta t_1}\right) - 1 + \frac{\Delta t_2}{\Delta t_1}\right] \quad (k)$$

The following algorithm can be used to obtain the optimal values of h_i, h_o, A_o, and Δt_2 without the explicit calculation of ω:

1. Solve for h_i from Equation (j)
2. Obtain h_o from Equation (i)
3. Calculate U_o from Equation (c)
4. Determine E_i and E_o from h_i and h_o using Equations (d) and (e) and obtain Δt_2 by solving Equation (k)
5. Calculate A_o from Equation (b)
6. Find W_c from Equation (f)

Note that steps 1 to 6 require that several nonlinear equations be solved one at a time. Once these variables are known, the physical dimensions of the heat exchanger can be determined.

7. Determine the optimal v_i and v_o from h_i and h_o using the appropriate heat transfer correlations (see McAdams, 1942); recall that the inside and outside tube diameters are specified a priori.
8. The number of tubes N_t can be found from a mass balance:

$$v_i N_t \frac{\pi D_i^2}{4} = W_i \quad (l)$$

9. The length of the tubes L_t can be found from

$$A_o = N_t \pi D_o L_t \quad (m)$$

10. The number of clearances N_c can be found from N_t, based on either square pitch or equilateral pitch. The flow area S_o is obtained from v_o (flow normal to a tube bundle). Finally, baffle spacing (or the number of baffles) is computed from S_o, A_o, N_t, and N_c.

Having presented the pertinent equations and the procedure for computing the optimum, let us check the approach by computing the degrees of freedom in the design problem.

Design Variables	*Status (number of variables)*
W_i, T_1, T_2, t_1, tube spacing, D_i, D_o, Q	Given (8)
Δt_2, W_c, A_o, N_t, L_t, U_o, n_b, Δp_t, Δp_s, v_i, v_o, h_i, h_o	Unspecified (13)

Total number of variables $= 8 + 13 = 21$

Design Relationships	*Number of Equations*
1. Equations (*b*), (*c*), (*d*), (*e*) (*f*), (*l*), (*m*)	7
2. Heat transfer correlations for h_i and h_o (step 7)	2
3. $W_c = \rho_o v_o s_o$ (step 10)	1
Total number of relationships	10

Degrees of freedom for optimization = total number of variables − number of given variables − number of equations

$$= 21 - 8 - 10 = 3$$

Note this result agrees with Equation (*h*) in that four variables are included in the Lagrangian, but with one constraint corresponding to 3 degrees of freedom.

Several simplified cases may be encountered in heat exchanger design.

Case 1. U_o is specified and pressure drop costs are ignored in the objective function. In this case C_i and C_o can be set equal to zero and Equation (*k*) can be solved for Δt_2 (see Peters and Timmerhaus (1980) for a similar equation for a condensing vapor). Figure E11.2b shows a solution to Equation (*k*) (Cichelli and Brinn).

Case 2. Coolant flow rate is fixed. Here Δt_2 is known, so the tube side and shell side coefficients and area are optimized. Use Equation (*i*) and (*j*) to find h_o and h_i. A_o is then found from Equation (*b*).

In the preceding analysis no inequality constraints were introduced. As a practical matter the following inequality constraints may apply:

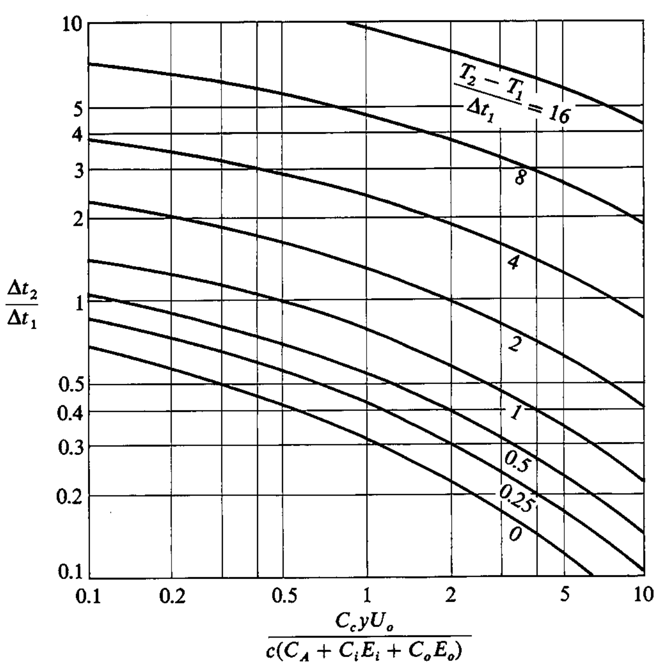

FIGURE E11.2b
Solution to Equation (*k*) for the case in which U_o is specified and pressure drop costs are ignored.

TABLE E11.2B
Design specifications for one case of heat exchanger optimization

Variables	
Process fluid	Gas
Inlet temperature of process fluid, °F	150
Outlet temperature of process fluid, °F	100
Process fluid flow rate, lb/h	20,000
Maximum process fluid velocity, ft/s	160
Minimum process fluid velocity, ft/s	0.001
Utility fluid	Water
Inlet utility fluid temperature, °F	70
Maximum allowable utility fluid temperature, °F	140
Maximum utility fluid velocity, ft/s	8
Minimum utility fluid velocity, ft/s	0.5
Shell side fouling factor	2000
Tube side fouling factor	1500
Cost of pumping process fluid, $/(ft)(lb$_f$)	0.7533×10^{-8}
Cost of pumping utility fluid, $/(ft)(lb$_f$)	0.7533×10^{-8}
Cost of utility fluid, $/lb$_m$	0.5000×10^{-5}
Factor for pressure	1.45
Cost index	1.22
Fractional annual fixed charges	0.20
Fractional cost of installation	0.15
Tube material	Steel
Type of tube layout	Triangular
Construction type	Fixed tube sheet
Maximum allowable shell diameter, in.	40
Bypassing safety factor	1.3
Constant for evaluating outside film coat	0.33
Hours operation per year	7000
Thermal conductivity of metal Btu/(h)(ft^2)(°F)	26
Number of tube passes	1

Source: Tarrer et al. (1971).

1. Maximum velocity on shell or tube side
2. Longest practical tube length
3. Closest practical baffle spacing
4. Maximum allowable pressure drops (shell or tube side)

The velocity on the tube side can be modified by changing the single-pass design to a multiple-pass configuration. In this case $F_t \neq 1$ in Equation (*b*). From formulas in McCabe, F_t depends on t_2 (or Δt_2), hence the necessary conditions derived previously would have to be changed. The fluids could be switched (shell vs. tube side) if constraints are violated, but there may well be practical limitations such as one fluid being quite dirty or corrosive so that the fluid must flow in the tube side (to facilitate cleaning or to reduce alloy costs).

Other practical features that must be taken into account are the fixed and integer lengths of tubes (8, 12, 16, and 20 feet), and the maximum pressure drops allowed.

TABLE 11.2C
Optimal solution for a heat exchanger involving discrete variables

Variables	Continuous-Variable Optimal Design	Standard integer sizes			
		1	2	3	4
Tube length, ft	10.5	8	8	12	12
Number of tubes	66	110	85	64	42
Total area, ft^2	193.3	230	178	201	132
Total cost, $/year	734	908	923	738	784
Heat transfer coefficients, Btu/(h)(ft^2)(°F)					
Outside	554	561	649	512	617
Inside	56.2	37.1	45.9	57.4	80.5
Overall	41.0	28.4	34.5	41.5	56.2
Outlet utility fluid temperature (°F)	117.1	102.1	96.5	120.1	112.4
Utility fluid flow rate, lb_m/h	5306	7790	9422	4993	5897
Inside pressure drop, psi	0.279	0.086	0.138	0.318	0.701
Outside pressure drop, psi	6.45	5.24	7.91	4.98	9.13
Number of baffle spaces	119	85	79	121	119
Shell diameter, in.	12	16	14	12	10

Tube layout: 1.00-in. outside diameter
0.834-in. inside diameter
0.25-in. clearance
0.083-in. wall thickness
1.25-in. pitch

Source: Tarrer et al. (1971).

Although a 20-psi drop may be typical for liquids such as water, higher values are employed for more viscous fluids. Exchanging shell sides with tube sides may mitigate pressure drop restrictions. The tube's outside diameter is specified a priori in the optimization procedure described earlier; usually $\frac{3}{4}$- or 1-inch outside diameter (o.d.) tubes are used because of their greater availability and ease of cleaning. Limits on operating variables, such as maximum exit temperature of the coolant, maximum and minimum velocities for both streams, and maximum allowable shell area must be included in the problem specifications along with the number of tube passes.

Table 11.2B lists the specifications for a typical exchanger, and Table 11.2C gives the results of optimization for several cases for two standard tube lengths, 8 and 12 ft. The minimum cost occurs for a 12-ft tube length with 64 tubes (case 3). Many commercial codes exist to carry out heat exchanger design. Search the Web for the most recent versions.

EXAMPLE 11.3 OPTIMIZATION OF A MULTI-EFFECT EVAPORATOR

When a process requires an evaporation step, the problem of evaporator design needs serious examination. Although the subject of evaporation and the equipment to carry out evaporation have been studied and analyzed for many years, each application has to receive individual attention. No evaporation configuration and its equipment can be picked from a stock list and be expected to produce trouble-free operation.

An engineer working on the selection of optimal evaporation equipment must list what is "known," "unknown," and "to be determined." Such analysis should at least include the following:

Known

- Production rate and analysis of product
- Feed flow rate, feed analysis, feed temperature
- Available utilities (steam, water, gas, etc.)
- Disposition of condensate (location) and its purity
- Probable materials of construction

Unknown

- Pressures, temperatures, solids, compositions, capacities, and concentrations
- Number of evaporator effects
- Amount of vapor leaving the last effect
- Heat transfer surface

Features to be determined

- Best type of evaporator body and heater arrangement
- Filtering characteristics of any solids or crystals
- Equipment dimensions, arrangement
- Separator elements for purity of overhead vapors
- Materials, fabrication details, instrumentation

Utility consumption

- Steam
- Electric power
- Water
- Air

In multiple-effect evaporation, as shown in Figure E11.3a, the total capacity of the system of evaporation is no greater than that of a single-effect evaporator having a heating surface equal to one effect and operating under the same terminal conditions. The amount of water vaporized per unit surface area in n effects is roughly $1/n$ that of a single effect. Furthermore, the boiling point elevation causes a loss of available temperature drop in every effect, thus reducing capacity. Why, then, are multiple effects often economic? It is because the cost of an evaporator per square foot of surface area decreases with total area (and asymptotically becomes a constant value) so that to achieve a given production, the cost of heat exchange surface can be balanced with the steam costs.

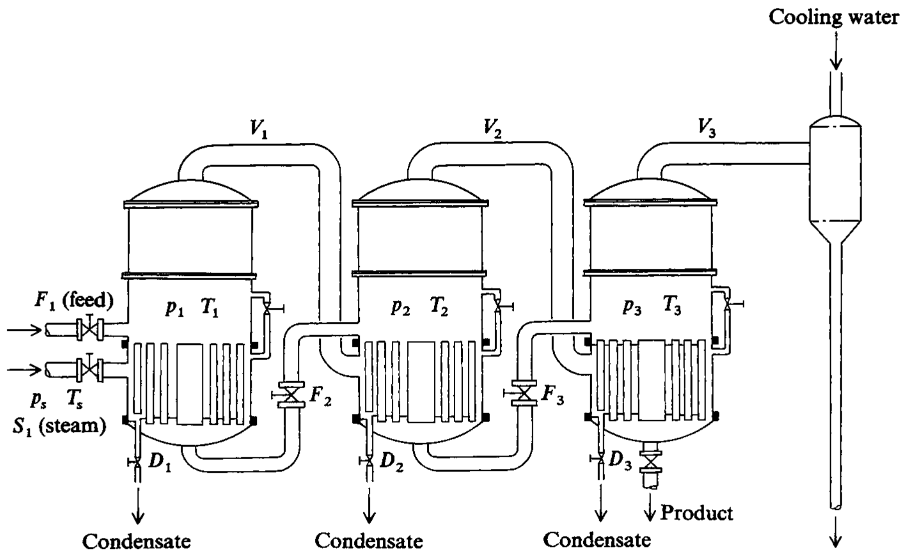

FIGURE E11.3a
Multiple-effect evaporator with forward feed.

Steady-state mathematical models of single- and multiple-effect evaporators involving material and energy balances can be found in McCabe et al. (1993), Yanniotis and Pilavachi (1996), and Esplugas and Mata (1983). The classical simplified optimization problem for evaporators (Schweyer, 1955) is to determine the most suitable number of effects given (1) an analytical expression for the fixed costs in terms of the number of effects n, and (2) the steam (variable) costs also in terms of n. Analytic differentiation yields an analytical solution for the optimal n^*, as shown here.

Assume we are concentrating an inorganic salt in the range of 0.1 to 1.0 wt% using a plant capacity of 0.1–10 million gallons/day. Initially we treat the number of stages n as a continuous variable. Figure E11.3b shows a single effect in the process.

Prior to discussions of the capital and operating costs, we need to define the temperature driving force for heat transfer. Examine the notation in Figure E11.3c; by definition the log mean temperature difference ΔT_{lm} is

$$\Delta T_{\text{lm}} = \frac{T_i - T_d}{\ln(T_i/T_d)} \qquad (a)$$

Let T_i be equal to constant K for a constant performance ratio P. Because $T_d = T_i - \Delta T_f/n$

$$\Delta T_{\text{lm}} = \frac{\Delta T_f/n}{\ln[K/K - (T_f/n))]} \qquad (b)$$

Let A = condenser heat transfer areas, ft^2

c_p = liquid heat capacity, 1.05 Btu/(lb$_{\text{m}}$)(°F)

C_C = cost per unit area of condenser, \$6.25/ft^2

C_E = cost per evaporator (including partitions), \$7000/stage

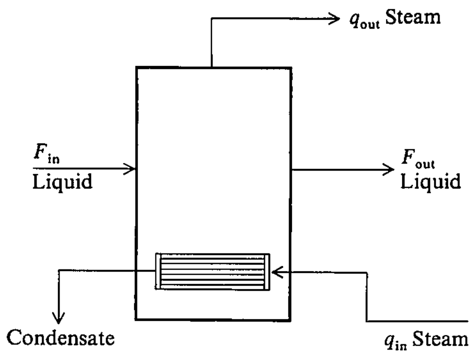

FIGURE E11.3b

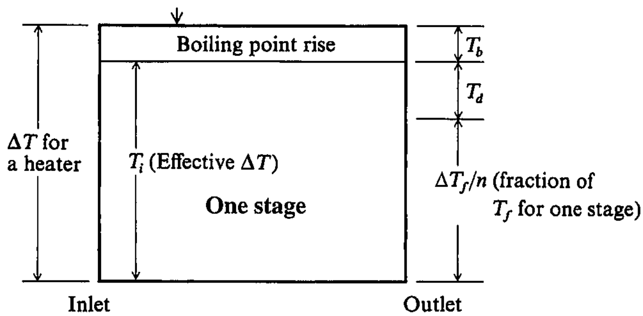

FIGURE E11.3c

C_S = cost of steam, \$/lb at the brine heater (first stage)
F_{out} = liquid flow out of evaporator, lb/h
$K = T_i$, a constant ($T_i = \Delta T - T_b$ at inlet)
n = number of stages
P = performance ratio, lb of H_2O evaporated/Btu supplied to brine heater
Q = heat duty, 9.5×10^8 Btu/h (a constant)
q_e = total lb H_2O evaporated/h
q_r = total lb steam used/h
r = capital recovery factor
S = lb steam supplied/h
T_b = boiling point rise, 4.3°F
ΔT_f = flash down range, 250°F
U = overall heat transfer coefficient (assumed to be constant), 625 Btu/(ft^2)(h)(°F)
ΔH_{vap} = heat of vaporization of water, about 1000 Btu/lb

The optimum number of stages is n^*. For a *constant performance ratio* the total cost of the evaporator is

$$f_1 = C_E n + C_C A \qquad (c)$$

For A we introduce

$$A = \frac{Q}{U(\Delta T_{\mathrm{lm}})}$$

Then we differentiate f_1 in Equation (c) with respect to n and set the resulting expression equal to zero (Q and U are constant):

$$C_E + C_C \frac{Q}{U}\left[\frac{\partial(1/\Delta T_{\mathrm{lm}})}{\partial n}\right]_P = 0 \tag{d}$$

With the use of Equation (b)

$$\left[\frac{\partial(1/\Delta T_{\mathrm{lm}})}{\partial n}\right]_P = -\frac{1}{nK(1 - \Delta T_f/nK)} - \frac{\ln(1 - \Delta T_f)}{\Delta T_f} \tag{e}$$

Substituting Equation (e) into (d) plus introducing the values of Q, U, ΔT_f, C_E, and C_C, we get

$$7000 - \left[\frac{(6.25)(9.5 \times 10^8)}{625}\right]\left[\frac{1}{nK(1 - \Delta T_f/nK)} + \frac{\ln(1 - \Delta T_f/nK)}{\Delta T_f}\right] = 0$$

Rearranging

$$\frac{(625)(7000)(250)}{(6.25)(9.5 \times 10^8)} = 0.184 = \frac{250}{nK - 250} + \ln\left(1 - \frac{250}{nK}\right) \tag{f}$$

In practice, as the evaporation plant size changes (for constant Q), the ratio of the stage condenser area cost to the unit evaporator cost remains essentially constant so that the number 0.184 is treated as a constant for all practical purposes. Equation (f) can be solved for nK for constant P

$$nK = 590 \tag{g}$$

Next, we eliminate K from Equation (g) by replacing K with a function of P so that n becomes a function of P. The performance ratio (with constant liquid heat capacity at 347°F) is defined as

$$P = \frac{(\Delta H_{\mathrm{vap}})(q_e)}{(F_{\mathrm{out}} c_{pF} \Delta T_{\mathrm{heater}})_{\text{first stage}}} = \frac{1000}{1.05(4.3 + K)} \frac{q_e}{F_{\mathrm{out}}} \tag{h}$$

The ratio q_e/F can be calculated from

$$\frac{q_e}{F_{\mathrm{out}}} = 1 - \left(\frac{1194 - 322}{1194 - 70}\right)^{1.49} = 0.31$$

where ΔH_{vap} (355°F, 143 psi) $= 1194$ Btu/lb
$\Delta H_{\mathrm{liq\ H_2O}}$ (350°F) $= 322$ Btu/lb
$\Delta H_{\mathrm{liq\ H_2O}}$ (100°F) $= 70$ Btu/lb

Equations (g) and (h) can be solved together to eliminate K and obtain the desired relation

$$\frac{300}{P} - 4.3 = \frac{590}{n^*} \tag{i}$$

Equation (i) shows how the boiling point rise ($T_b = 4.3°\text{F}$) and the number of stages affects the performance ratio.

Optimal performance ratio

The optimal plant operation can be determined by minimizing the total cost function, including steam costs, with respect to P (liquid pumping costs are negligible)

$$f_2 = [C_c A + C_E n]r + C_s S \tag{j}$$

$$rC_C \frac{\partial A}{\partial P} + rC_E \frac{\partial n}{\partial P} + C_s \frac{\partial S}{\partial P} = 0 \tag{k}$$

The quantity for $\partial A/\partial P$ can be calculated by using the equations already developed and can be expressed in terms of a ratio of polynomials in P such as

$$\frac{a(1 + 1/P)}{(1 - bP)^2}$$

where a and b are determined by fitting experimental data. The relation for $\partial n/\partial P$ can be determined from Equation (i). The relation for $\partial S/\partial P$ can be obtained from equation (l)

$$P = \frac{q_e}{Q} = \frac{q_e}{(\Delta H_{\text{vap}})S} = \frac{q_e}{1000S}$$

or

$$S\left(\frac{\text{lb}}{\text{h}}\right) = \frac{q_e}{1000P}$$

or

$$S(\text{lb}) = \frac{\alpha(8760)q_e}{1000P} \tag{l}$$

where α is the fraction of hours per year (8760) during which the system operates.

Equation (k), given the costs, cannot be explicitly solved for P^*, but P^* can be obtained by any effective root-finding technique.

If a more complex mathematical model is employed to represent the evaporation process, you must shift from analytic to numerical methods. The material and enthalpy balances become complicated functions of temperature (and pressure). Usually all of the system parameters are specified except for the heat transfer areas in each effect (n unknown variables) and the vapor temperatures in each effect excluding the last one ($n - 1$ unknown variables). The model introduces n independent equations that serve as constraints, many of which are nonlinear, plus nonlinear relations among the temperatures, concentrations, and physical properties such as the enthalpy and the heat transfer coefficient.

Because the number of evaporators represents an integer-valued variable, and because many engineers use tables and graphs as well as equations for evaporator calculations, some of the methods outlined in Chapters 9 and 10 can be applied for the optimization of multi-effect evaporator cascades.

EXAMPLE 11.4 BOILER/TURBO-GENERATOR SYSTEM OPTIMIZATION

Linear programming is often used in the design and operation of steam systems in the chemical industry. Figure E11.4 shows a steam and power system for a small power house fired by wood pulp. To produce electric power, this system contains two turbo-generators whose characteristics are listed in Table E11.4A. Turbine 1 is a double-extraction turbine with two intermediate streams leaving at 195 and 62 psi; the final stage produces condensate that is used as boiler feed water. Turbine 2 is a single-

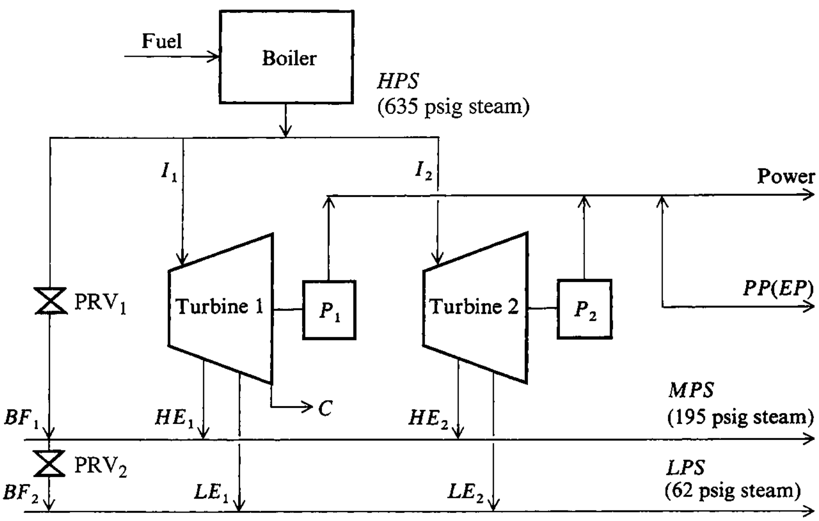

FIGURE E11.4
Boiler/turbo-generator system.
Key: I_i = inlet flow rate for turbine i [lb_m/h]
HE_i = exit flow rate from turbine i to 195 psi header [lb_m/h]
LE_i = exit flow rate from turbine i to 62 psi header [lb_m/h]
C = condensate flow rate from turbine 1 [lb_m/h]
P_i = power generated by turbine i [kW]
BF_1 = bypass flow rate from 635 psi to 195 psi header [lb_m/h]
BF_2 = bypass flow rate from 195 psi to 62 psi header [lb_m/h]
HPS = flow rate through 635 psi header [lb_m/h]
MPS = flow rate through 195 psi header [lb_m/h]
LPS = flow rate through 62 psi header [lb_m/h]
PP = purchased power [kW]
EP = excess power [kW] (difference of purchased power from base power)
PRV = pressure-reducing valve

extraction turbine with one intermediate stream at 195 psi and an exit stream leaving at 62 psi with no condensate being formed. The first turbine is more efficient due to the energy released from the condensation of steam, but it cannot produce as much power as the second turbine. Excess steam may bypass the turbines to the two levels of steam through pressure-reducing valves.

Table E11.4B lists information about the different levels of steam, and Table E11.4C gives the demands on the system. To meet the electric power demand, electric power may be purchased from another producer with a minimum base of 12,000 kW. If the electric power required to meet the system demand is less than this base, the power that is not used will be charged at a penalty cost. Table E11.4D gives the costs of fuel for the boiler and additional electric power to operate the utility system.

The system shown in Figure E11.4 may be modeled as linear constraints and combined with a linear objective function. The objective is to minimize the operating cost of the system by choice of steam flow rates and power generated or purchased, subject to the demands and restrictions on the system. The following objective function is the cost to operate the system per hour, namely, the sum of steam produced *HPS*, purchased power required *PP*, and excess power *EP*:

TABLE 11.4A
Turbine data

Turbine 1		Turbine 2	
Maximum generative capacity	6,250 kW	Maximum generative capacity	9,000 kW
Minimum load	2,500 kW	Minimum load	3,000 kW
Maximum inlet flow	192,000 lb_m/h	Maximum inlet flow	244,000 lb_m/h
Maximum condensate flow	62,000 lb_m/h	Maximum 62 psi exhaust	142,000 lb_m/h
Maximum internal flow	132,000 lb_m/h	High-pressure extraction at	195 psig
High-pressure extraction at	195 psig	Low-pressure extraction at	62 psig
Low-pressure extraction at	62 psig		

TABLE 11.4B
Steam header data

Header	Pressure (psig)	Temperature (°F)	Enthalpy (Btu/lb_m)
High-pressure steam	635	720	1359.8
Medium-pressure steam	195	130 superheat	1267.8
Low-pressure steam	62	130 superheat	1251.4
Feedwater (condensate)			193.0

TABLE 11.4C
Demands on the system

Resource	Demand
Medium-pressure steam (195 psig)	271,536 lb_m/h
Low-pressure steam (62 psig)	100,623 lb_m/h
Electric power	24,550 kW

TABLE 11.4D
Energy data

Fuel cost	\$1.68/10^6 Btu
Boiler efficiency	0.75
Steam cost (635 psi)	\$2.24/10^6 Btu
	= \$2.24 (1359.8 − 193)/10^6
	= \$0.002614/lb_m
Purchased electric power	\$0.0239/kWh average
Demand penalty	\$0.009825/kWh
Base-purchased power	12,000 kW

$$\text{Minimize:} \quad f = 0.00261\, HPS + 0.0239\, PP + 0.00983\, EP \tag{a}$$

The constraints are gathered into the following specific subsets:

Turbine 1

$$\begin{aligned} P_1 &\le 6250 \\ P_1 &\ge 2500 \\ HE_1 &\le 192{,}000 \\ C &\le 62{,}000 \\ I_1 - HE_1 &\le 132{,}000 \end{aligned} \tag{b}$$

Turbine 2

$$\begin{aligned} P_2 &\le 9000 \\ P_2 &\ge 3000 \\ I_2 &\le 244{,}000 \\ LE_2 &\le 142{,}000 \end{aligned} \tag{c}$$

Material balances

$$\begin{aligned} HPS - I_1 - I_2 - BF_1 &= 0 \\ I_1 + I_2 + BF_1 - C - MPS - LPS &= 0 \\ I_1 - HE_1 - LE_1 - C &= 0 \\ I_2 - HE_2 - LE_2 &= 0 \\ HE_1 + HE_2 + BF_1 - BF_2 - MPS &= 0 \\ LE_1 + LE_2 + BF_2 - LPS &= 0 \end{aligned} \tag{d}$$

Power purchased

$$EP + PP \ge 12{,}000 \tag{e}$$

Demands

$$\begin{aligned} MPS &\geq 271{,}536 \\ LPS &\geq 100{,}623 \\ P_1 + P_2 + PP &\geq 24{,}550 \end{aligned} \qquad (f)$$

Energy balances

$$\begin{aligned} 1359.8I_1 - 1267.8HE_1 - 1251.4LE_1 - 192C - 3413P_1 &= 0 \\ 1359.8\, I_2 - 1267.8\, I_2 - 1251.4\, LE_2 - 3413\, P_2 &= 0 \end{aligned} \qquad (g)$$

TABLE E11.4E
Optimal solution to steam system LP

Variable	Name	Value	Status
1	I_1	136,329	BASIC
2	I_2	244,000	BOUND
3	HE_1	128,158	BASIC
4	HE_2	143,377	BASIC
5	LE_1	0	ZERO
6	LE_2	100,623	BASIC
7	C	8,170	BASIC
8	BF_1	0	ZERO
9	BF_2	0	ZERO
10	HPS	380,329	BASIC
11	MPS	271,536	BASIC
12	LPS	100,623	BASIC
13	P_1	6,250	BOUND
14	P_2	7,061	BASIC
15	PP	11,239	BASIC
16	EP	761	BASIC

Value of objective function = 1268.75 \$/h
BASIC = basic variable
ZERO = 0
BOUND = variable at its upper bound

Table E11.4E lists the optimal solution to the linear program posed by Equations (a)–(g). Basic and nonbasic (zero) variables are identified in the table; the minimum cost is \$1268.75/h. Note that $EP + PP$ must sum to 12,000 kWh; in this case the excess power is reduced to 761 kWh.

REFERENCES

Athier, G.; P. Floquet; L. Pibouleau; et al. "Process Optimization by Simulated Annealing and NLP Procedures. Application to Heat Exchanger Network Synthesis." *Comput Chem Eng* **21 (Suppl):** S475–S480 (1997).

Briones, V.; and A. Kokossis. "A New Approach for the Optimal Retrofit of Heat Exchanger Networks." *Comput Chem Eng* **20 (Suppl):** S43–S48 (1996).

Cichelli, M. T.; and M. S. Brinn. "How to Design the Optimum Heat Exchanger." *Chem Eng* **196:** May (1956).

Esplugas, S.; and J. Mata. "Calculator Design of Multistage Evaporators." *Chem Eng* **59** Feb. 7: (1983).

Jegede, F. O.; and G. T. Polley. "Capital Cost Targets for Networks with Non-Uniform Heat Transfer Specifications." *Comput Chem Eng* **16:** 477 (1992).

McAdams, W. H. *Heat Transmission.* McGraw-Hill, New York (1942).

McCabe, W. L.; J. Smith; and P. Harriott. *Unit Operations in Chemical Engineering*, 5th ed. McGraw-Hill, New York (1993).

Peters, M.; and K. Timmerhaus. *Plant Design and Economics for Chemical Engineers*, 4th ed. McGraw-Hill, New York (1991).

Reppich, M.; and S. Zagermann. "A New Design Method for Segementally Baffled Heat Exchangers." *Comput Chem Eng* **19 (Suppl):** S137–S142 (1995).

Sama, D. A. "Economic Optimum LMTD at Heat Exchangers." *AIChE National Meeting.* Houston, Texas (March 1983).

Schweyer, H. E. *Process Engineering Economics.* McGraw-Hill, New York, (1955), p. 214.

Steinmeyer, D. E. "Process Energy Conservation." *Kirk-Othmer Encyclopedia* Supplemental Volume, 3d ed. Wiley, New York (1984).

Swearingen, J. S.; and J. E. Ferguson. "Optimized Power Recovery from Waste Heat." *Chem Eng Prog* August 66-70 (1983).

Tarrer, A. R.; H. C. Lim; and L. B. Koppel. "Finding the Economically Optimum Heat Exchanger." *Chem Eng* **79:** 79–84 Oct. 4 (1971).

Yanniotis, S.; and P. A. Pilavachi. "Mathematical Modeling and Experimental Validation of an Absorber-Driven Multiple Effect Evaporator." *Chem Eng Technol* **19:** 448–455 (1996).

Zamora, J. M.; and I. E. Grossmann. "A Global MINLP Optimization Algorithm for the Synthesis of Heat Exchanger Networks with No Stream Splits." *Comput Chem Eng* **22:** 367–384 (1998).

SUPPLEMENTARY REFERENCES

Chaudhuri, P. D.; U. M. Diweker; and J. S. Logsdon. "An Automated Approach for the Optimal Design of Heat Exchangers." *Ind Eng Chem Res* **36** (9): 3685–3693 (1999).

Ciric, A. R.; and C. A. Floudas. "Heat Exchanger Network Synthesis Without Documentation." *Comput Chem Eng* **15:** 385–396 (1991).

Colmenares, T. R.; and W. D. Seider. "Heat and Power Integration of Chemical Processes." *AIChE J* **33:** 898–915 (1987).

Cornellisen, R. L.; and G. G. Hiss. "Thermodynamic Optimization of a Heat Exchanger." *Int J Heat Mass Transfer* **42** (5): 951–959 (1999).

Daichendt, M. M.; and I. E. Grossmann. "Preliminary Screening Procedure for the MINLP Synthesis of Process Systems II. Heat Exchanger Networks." *Comput Chem Eng* **18:** 679–710 (1994).

Duran, M. A.; and I. E. Grossmann. "Simultaneous Optimization and Heat Integration of Chemical Processes." *AIChE J* **32:** 123–138 (1986).

Fabbri, G. "Heat Transfer Optimization in Internally Finned Tubes Under Laminar Flow Conditions." *Int J Heat Mass Transfer* **41** (10): 1243–1253 (1998).

Georgiadis, M. C.; L. G. Papageorgiou; and S. Macchietto. "Optimal Cleaning Policies in Heat Exchanger Networks under Rapid Fouling." *Ind Eng Chem Res* **39** (2): 441–454 (2000).

Gunderson, T.; and L. Naess. "The Synthesis of Cost Optimal Heat Exchanger Networks. An Industrial Review of the State of the Art." *Comput Chem Eng* **12:** 503–530 (1988).

Ikegami, Y.; and A. Bejan. "On the Thermodynamic Optimization of Power Plants with Heat Transfer and Fluid Flow Irrreversibilities." *J Solar Energy Engr* **120** (2): 139–144 (1998).

Kalitventzeff, B. "Mixed Integer Nonlinear Programming and its Application to the Management of Utility Networks." *Eng Optim* **18:** 183–207 (1991).

Lang, Y. D.; L. T. Biegler; and I. E. Grossmann. "Simultaneous Optimization and Heat Integration with Process Simulators." *Comput Chem Eng* **12:** 311–328 (1988).

Linnhoff, B. "Pinch Analysis—A State-of-the-Art Overview." *Trans I Chem E* **71** (A): 503– 523 (1993).

Luus, R. "Optimization of Heat Exchanger Networks." *Ind Eng Chem Res* **32** (11): 2633– 2635 (1993).

Peterson, J.; and Y. Bayazitoglu. "Optimization of Cost Subject to Uncertainty Constraints in Experimental Fluid Flow and Heat Transfer." *J Heat Transfer* **113:** 314–320 (1991).

Quesada, I.; and I. E. Grossmann. "Global Optimization Algorithm for Heat Exchanger Networks." *Ind Eng Chem Res* **32:** 487–499 (1993).

Tayal, M. C.; Y. Fu; and U. M. Diwekar. "Optimal Design of Heat Exchangers: A Genetic Algorithm Framework." *Ind Eng Chem Res* **38** (2): 456–467 (1999).

Yee, T. F.; I. E. Grossmann; and Z. Kravanja. "Simultaneous Optimization Models for Heat Integration—I. Area and Energy Targeting and Modeling of Multistream Exchangers." *Comput Chem Eng* **14:** 1165–1183 (1990).

Yeh, R. "Errors in One-dimensional Fin Optimization Problem for Convective Heat Transfer." *Int J Heat Mass Transfer* **39** (14): 3075–3078 (1996).

Zhu, X. X.; and N. D. K. Asante. "Diagnosis and Optimization Approach for Heat Exchanger Network Retrofit." *AIChE J* **45:** 1488–1503 (1999).