EXPERIMENTAL AIR-PRESSURE TANK SYSTEMS

for Process Control Education

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Process control education is a significant aspect of the chemical engineering curriculum, as it provides a fundamental basis for modern chemical process operation. The subject is highly applied yet rooted deeply in theory. Bridging the gap between the theory and application is often a difficult task, particularly in the classroom setting. Experimental laboratories have been shown to be useful in motivating students and reinforcing the information taught in the classroom,[1-4] often with the additional benefit of small-group learning.[5,6] The use of hands-on experimental laboratories that are closely tied to the traditional process control lecture course allows students to actually link the theoretical content of the courses to its use on real-world systems. For this reason, process control experiments have been developed across the country.[7-9]

The development of useful, dynamic, process control experiments requires a number of considerations. Safety is the primary consideration because an environmentally friendly system which can be operated with minimal risk to both the equipment and the user is necessary. The ideal system would also be a cost-effective means to demonstrate the pertinent material with some industrial relevance. It should be of moderate complexity, as simple systems may be too trivial to motivate students while a full-scale industrial process may be too overwhelming. Giving it flexible configuration options will allow for its use in a variety of contexts. Reasonable process time constants are also essential so that the system dynamics are slow enough to demonstrate that process changes are not instantaneous, while also reacting quickly enough to limit student boredom when examining dynamic process transitions.

Undergraduate students typically have very limited experience with dynamic systems since many undergraduate courses work under assumptions of steady-state operation. The use of the dynamic experiment(s) provides this experience and demonstrates all aspects of the textbook theory.[10-17]

There are a number of well-designed, low-cost experiments available commercially, from vendors such as Lego, for use

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in process control education.[18] These systems, however, fail to offer the flexibility to be utilized in many different contexts. Furthermore, they often fail to provide any semblance of being industrially relevant.

At the University of South Carolina, both a simple, dynamic, nonlinear, two-tank, air-pressure system and a more complex multivariable, four-tank, air-pressure system have been developed. These pressure-tank systems prove quite useful in process control education, as they address the objectives for an ideal process control experiment. Inspired by experimental liquid-level systems,[19-23] these experiments are exceptional instructional tools for chemical engineers. As opposed to liquid-level systems, in these systems pressure differences drive the flow. This variation removes the limitations in system flexibility typically associated with gravity-driven liquid systems. The two-tank system is quite portable, thus lending itself well to classroom *and* outreach demonstrations. A variety of undergraduate topics including open-loop modeling and traditional single-input, single-output (SISO) closed-loop control strategies can be readily demonstrated on the two-tank system. The more complex multivariable, four-tank system can be used in a small group setting to illustrate more advanced topics such as multi-input, multi-output (MIMO) modeling, interacting systems, and multivariable decoupling, to name a few. This paper presents a detailed description of both systems and summarizes their current and future uses for both educational and research purposes.

THE TWO-TANK SYSTEM

A compact, experimental, air-pressure tank system involving a pair of tanks in series has been developed (<http://www.che.sc.edu/faculty/gatzke/software.htm>). A schematic and photograph of the system are provided in Figures 1 and 2. This section describes the system itself as well as presenting its uses in the context of undergraduate process control education.

System Description

The two-tank pressure system is comprised of two constant-volume aluminum tanks assembled in series supported by aluminum framework (22 inches long × 24 inches high × 17 inches wide). The two cylindrical tanks are each a foot in length. Their diameters are two inches and one inch, respectively. Supply air enters the system through a single one-half-inch, air-actuated, BadgerMeter control valve.[24] The air flows through quarter-inch tubing into the two tanks in series and exits to the atmosphere. A small muffler is utilized at the exit to reduce system noise. The tanks are separated by Swagelok[25] metering valves with repeatable vernier handles. This provides a means to accurately transform the system between various system configurations. Note that completely opening a valve between the two tanks effectively "joins" the tanks, resulting in one large tank of uniform pressure, as opposed to two tanks in series. Pressure measurements are available from each of the two pressure tanks. Gauges are installed on each tank to provide visual indications of the pressures while pressure transducers are used to more accurately measure and transmit pressure readings to a computer. The larger tank is also fitted with a small release valve that vents to the atmosphere. This valve can be used to create a disturbance on the system that might simulate a leak in the given tank, providing the opportunity to examine disturbance rejection as a possible control objective in addition to refer-

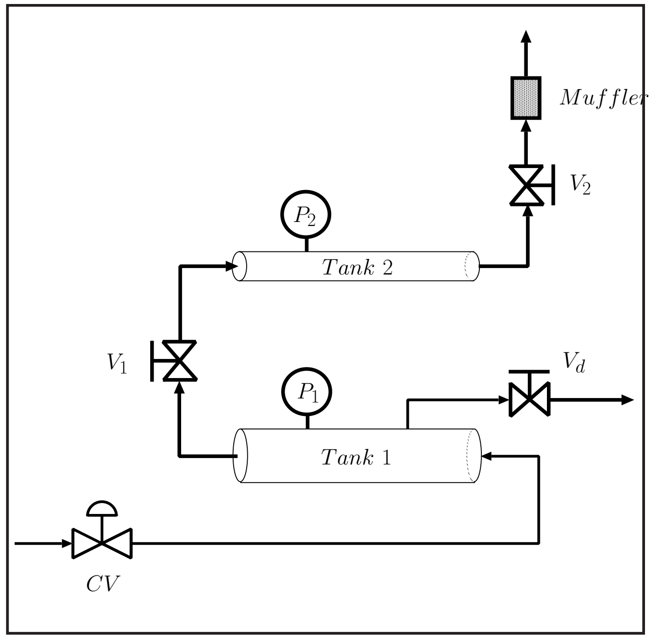

Figure 1. *Two -tank schematic.*

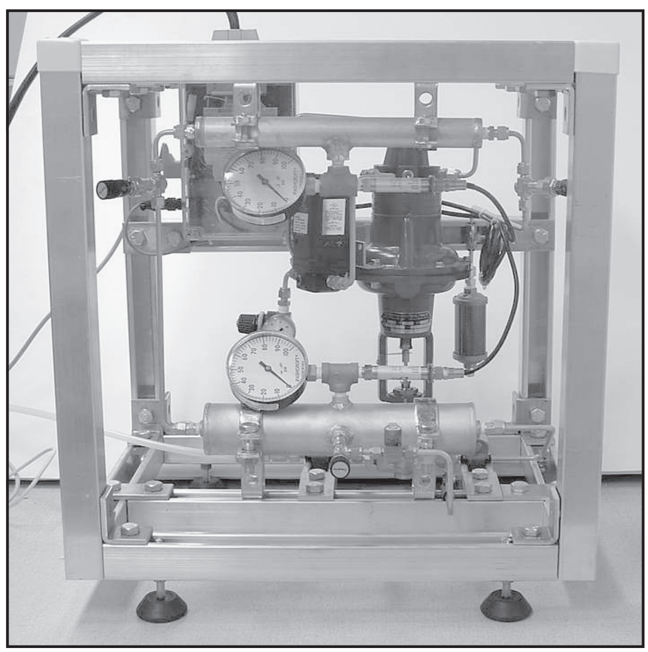

Figure 2. *Photograph of the two-tank system.*

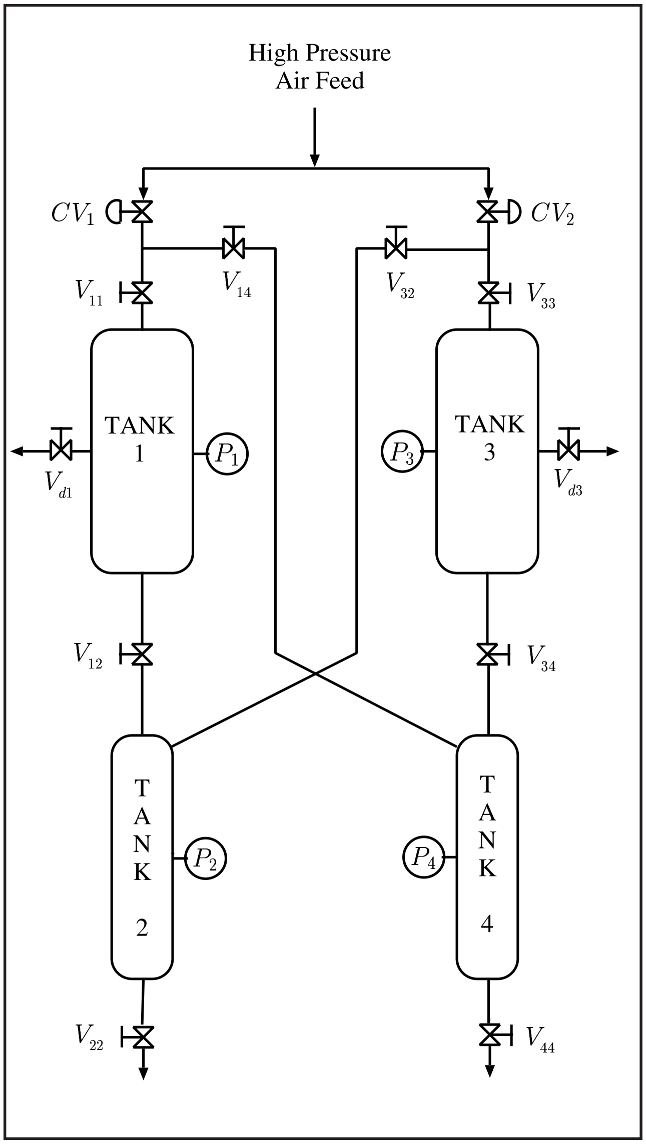

***Figure 3.** Schematic of the four-tank pressure system.*

ence tracking. The apparatus is equipped with a National Instruments Data Acquisition system which can be interfaced to both Matlab/Simulink[26] and LabView.[27] A complete materials listing can be obtained by contacting the authors.

It should be noted that initially the control valve exhibited substantial hysteresis, making accurate modeling impossible. A valve positioner was required in order to generate reproducible open-loops results on the system. This also helps introduce students to cascade control and the complexity of real industrial systems.

In the lab environment, the feed air pressure can be supplied in a more permanent manner from a compressor. On the other hand, small compressed-gas cylinders or lecture bottles can be used so that the system can be taken into the classroom for demonstrations. Similarly, a dedicated desktop computer can be used in the labs, while a laptop can be conveniently carried to the classroom.

Educational Uses

This new experimental system is quite valuable for educational purposes. In the classroom setting, it lends itself well for demonstration to larger audiences. Alternatively, smaller groups can experiment with the system in a laboratory setting and reap the benefits of learning in a "hands-on" environment. The typical undergraduate class can be broken into small groups that can be rotated between the actual pressure-tank system and nearby computer labs. In the computer labs, students can utilize a high-fidelity model of the system to carry out simulation work that closely parallels what is to be done experimentally. This way, those entering the computer labs first can prepare for the actual experiment, while those that see the actual system first can later reaffirm what has been done experimentally. These advantages are supported by the rapid dynamics of the system. Note that the open-loop time constant is on the order of 30 seconds. In an extended class period, it is possible that numerous groups could get a substantial amount of time working with the apparatus.

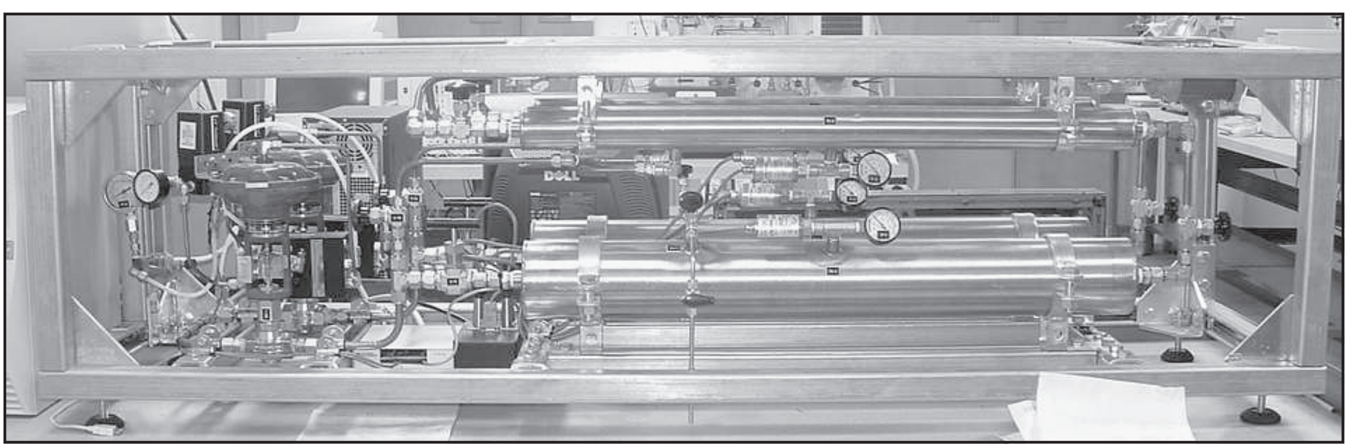

***Figure 4.** Photograph of the four-tank pressure system.*

Using this system, many topics from the undergraduate process control curriculum can be illustrated. Open-loop modeling can be performed to identify both first- and second order SISO models of the two tanks, depending on the configuration. Both frequency- and time-domain models can be considered, including input/output descriptions such as Autoregressive Moving Average (ARMA) models. Linearization of an available nonlinear first-principles model can also be carried out. Traditional closed-loop control methodologies such as Proportional-Integral-Derivative (PID) and Internal Model Control (IMC) can be implemented. Additionally, related topics such as closed-loop stability can be demonstrated.

THE FOUR-TANK SYSTEM

This section describes the four-tank system in comparison to the two-tank apparatus. A schematic and photograph of the system are provided in Figures 3 and 4. This system's uses for undergraduate, intermediate, and advanced process controls education are presented along with its utility in process systems engineering research.

System Description

The MIMO experimental system consists of four interconnected air tanks arranged in two parallel trains of two tanks, in series, built upon a steel framework. Each tank is 35 inches in length with diameters of 4 inches and 2.5 inches for the upstream and downstream tanks, respectively. Supply air flows into the system through two air-actuated BadgerMeter control valves which serve as the manipulated variables for the system. The air flows through copper tubing and the tanks before exiting to the atmosphere. Again, mufflers have been installed at the system exit to reduce the noise level. Specifically, the air flowing through control valve 1 (CV_1) proceeds into tank 1 and subsequently into tank 2 downstream before exiting the system. Additionally, a portion of the flow from the control valve can be routed into the downstream tank of the adjacent train (tank 4). In a similar manner, control valve 2 (CV_2) affects the pressure in tanks 3 and 4, with cross-flow effects on tank 2. Valves V_{14} and V_{32} are directly responsible for the cross-train flow. In some cases, the interacting nature of the system as a result of the cross-train flow leads to the presence of an adjustable, multivariable, right-half plane zero and inverse response. Physically, this is a result of the fast and direct response of the downstream tank pressures to cross-train flow, in contrast to the slow indirect effects of the flow from the large upstream tanks into the smaller downstream tanks.

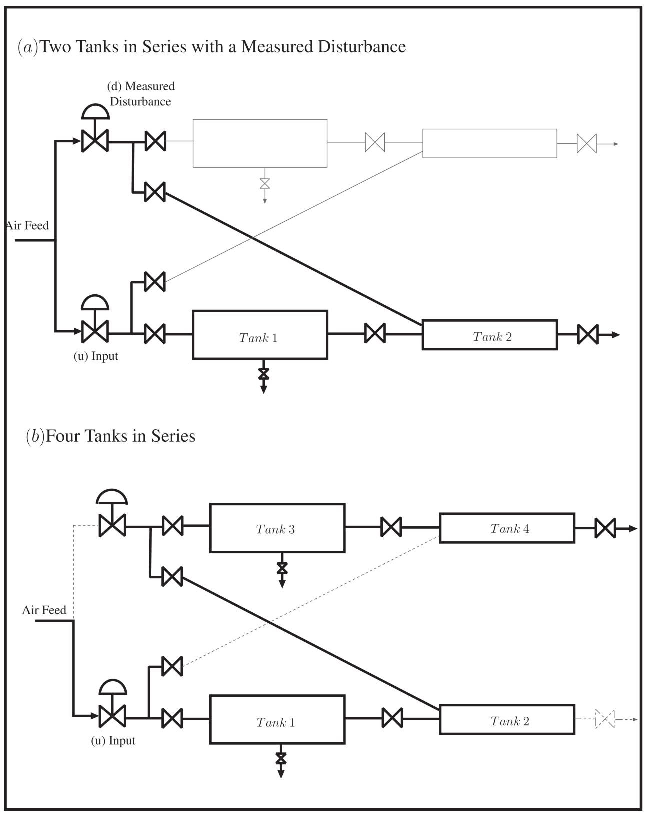

***Figure 5.** Flow diagram for alternative configurations of the four tank system.*

The flow of air through the system is driven by pressure gradients. Check valves are not used, therefore air could flow back upstream provided that the pressure gradient is in the appropriate direction. (Similar liquid levels have limitations in these regards as the flow path is dictated by gravity.) The result is a more flexible, dynamic experiment. As with the two-tank system, the various tanks are separated by a number of Swagelok metering valves; their placement allows the system to be configured in a variety of ways. By opening or closing select valves between the tanks, the system can be quickly transformed from one configuration to another. The possible configurations include: a single tank of numerous possible sizes (depending on the number of tanks utilized), two to four tanks in series, a pair of tanks in parallel, and other setups that would have tanks in both series and parallel. For example, V_{14}, V_{22}, and CV_2 can be completely closed, resulting in an SISO fourth-order system with air flowing through all tanks in series (see Figure 5b). Note that in the interest of saving laboratory space, the system is "folded" so that the smaller tanks are placed above the larger ones, leaving a system with total dimensions of 72 inches long, 22 inches high and 22 inches wide.

Educational Uses

Although not portable enough to be taken to the classroom, this system is well suited for use in the laboratory environment. This apparatus can again be used for large group demonstrations or in a more personal setting for individual-to-small-group work (see Figure 6).

The multivariable, four-tank pressure system can be configured in such a manner that it closely mimics the operation of the simple two-tank system, thus allowing one to demonstrate similar concepts. The additional complexity and flexibility of the four-tank system, however, also allow for its use in a wider variety of contexts, particularly with regard to its multivariable nature. The system can be configured such that one control valve acts as a measured disturbance into the downstream tank—thus allowing for feedforward control. This configuration is shown in Figure 5a. Input/output modeling of multiple tanks in series can be carried out given the appropriate configuration, but MIMO modeling techniques such as continuous and discrete-time, linear time-invariant (LTI), state-space approaches can also be applied. Interacting systems can be demonstrated as well as dynamic decoupling. The simulink interface showing PI control of the four-tank system is shown in Figure 7. In this feedback arrangement, the two downstream tank pressures are being controlled by manipulating the two control valves at the inlet. The disturbance rejection capabilities of this control scheme can be shown by simulating a leak in either of the upstream tanks or by changing the supply air pressure.

Figure 6. *Students performing lab on the tank system.*

In addition to aiding in the presentation and reinforcement of the undergraduate material, more advanced undergraduate and graduate topics can be covered using this sytem. Linear and nonlinear state and parameter estimation routines can be developed for the system. Advanced control schemes can be used including multivariable IMC, H_∞, and linear Model Predictive Control (MPC). With some tank configurations, the system can exhibit a multivariable right-half plane zero thus inverse response—motivating the examination of input directionality and control performance limitations.[16]

Student Assignments

For illustrative purposes, two relevant assignments typically given to the students in the undergraduate and advanced (intermediate and graduate-level) courses are provided.

- ***Undergraduate Assignment***

  Configure the four-tank system into an SISO arrangement that involves two tanks in series. Develop a transfer function representation of the relationship between the control valve and the pressure of the downstream tank. Using this model, implement an Internal Model Control (IMC) scheme on the system in Matlab/Simulink and test the closed-loop performance of the system by introducing both setpoint changes and disturbances.

- ***Advanced Assignment***

  Configure the four-tank system into a 2-by-2 MIMO arrangement that involves two parallel trains of two tanks in series with cross flow. Consider the two downstream tanks as process outputs and the two control valves as the manipulated variables. Utilize subspace identification methods in Matlab to develop a linear state-space representation of the system. Using this model, implement a traditional Model

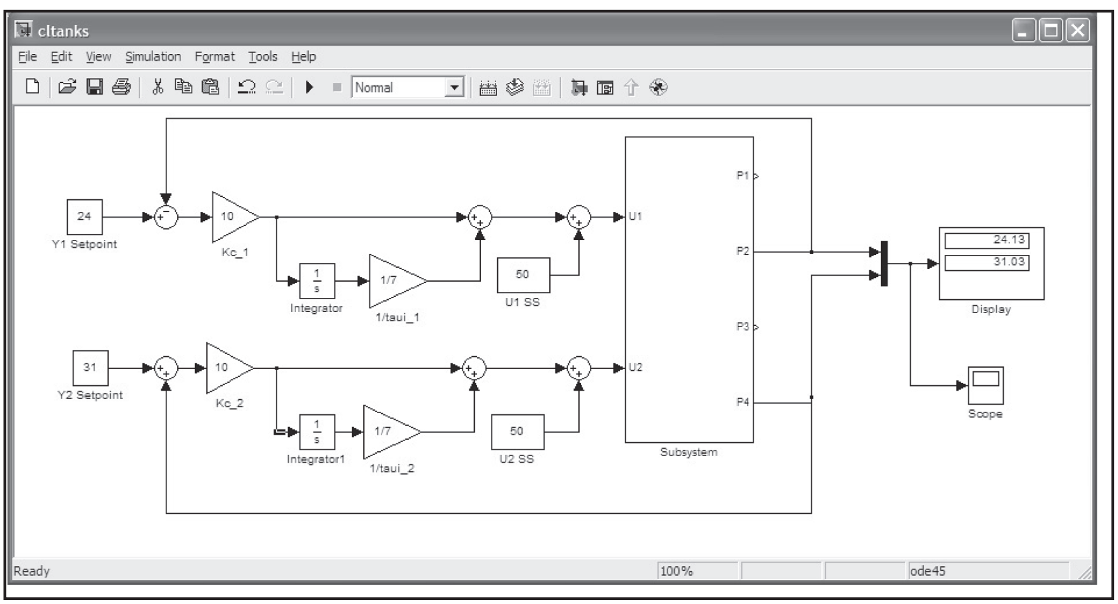

Figure 7. *Simulink Interface showing closed-loop control of the four-tank system.*

AUTHOR: YOU NEED TO MENTION FIG. 8 IN THE TEXT, ***BRIEFLY,*** on this page

Predictive Controller (MPC) on the system and test the closed-loop performance of the system by introducing both setpoint changes and disturbances. Test the impact of the various tuning parameters on the stability and performance of the controller.

These assignments exemplify those used in the different control courses. They provide students with the opportunity to explore the modeling and control the experimental pressure-tank system. Again, note that in the interest of time, some students can develop their control methodology using a high-fidelity process model as the system to be controlled before implementing their work on the actual system.

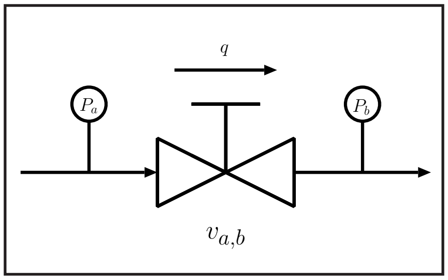

Figure 8. *Schematic showing the relationship of the pressures involved in calculating the gas flowrate across a valve. Adapted from Reference 25.*

Related Research

In addition to its utility in the instruction of process control theory, this four-tank system has potential for use in research in the field of systems engineering. To date, this particular system has been the focus of a number of research endeavors.

For instance, system modeling is an important precursor to many advanced model-based control schemes. In limited regions of operation a simple linear model could suffice. Process nonlinearities, however, often require more complex model forms. The nature of this system is such that the process can exhibit hybrid dynamic behavior as the flow of air through the valves of the system can discretely switch between distinct, multiple, continuous regimes of operation. Under low pressure-drop conditions, the air flowrate across a given valve is dependent on both the up- and downstream pressures. In high pressure-drop conditions, however, a sonic, or choke, flow regime is encountered in which the flowrate across a valve becomes solely dependent on the upstream pressure. The respective valve manufacturers, Swagelok[25] and BadgerMeter,[24] provide "hybrid" flow expressions based on first principles to capture these dynamics. For the BadgerMeter control valves the flow can be described by:

$$q = NC_v \sqrt{\frac{P_a \Delta P}{G_g T_a}} \quad \text{if } P_b \rangle 0.5P_a$$

or

$$q = NC_v \sqrt{\frac{(3/2)P_s^2}{G_g T_a}} \quad \text{if } P_b \le 0.5P_a$$

while for the Swagelok needle valve the flows can be described by:

$$q = NC_v P_a \left(1 - \frac{2\Delta P}{3P_a}\right) \sqrt{\frac{\Delta P}{P_a G_g T_a}} \quad \text{if } P_b \rangle 0.5P_a \qquad (1)$$

or

$$q = 0.471NC_v P_a \sqrt{\frac{1}{G_g T_a}} \quad \text{if } P_b \le 0.5P_a \qquad (2)$$

where q is a volumetric air flow rate across the valve at standard conditions, N is a numerical constant for units, C_v is the valve coefficient, P_a is the upstream pressure, G_g is the specific gravity of the fluid, and T_a is the temperature of the system. Temperature measurements are not available at the various points in the system. For convenience it is assumed that the temperature of the air in the system is approximately constant throughout. The first flow expression defines the low pressure drop regime where the flo across the valve is a function of both the upstream and downstream pressures. The second flow expression defines the choked flow regime where the downstream pressure has no influence on the flowrate. Under ideal conditions, these flow expressions can be used in conjunction with the ideal gas law to develop discrete-time models of the pressure in each tank.

To model the rate of change of pressure in a given tank $(\dot{P}_i)$, the ideal gas law is assumed as the system is operated at both a reasonable temperature and pressure.

$$\dot{P}_i = \frac{\dot{n}_i RT}{V_i} \qquad (3)$$

where V_i is the volume of the tank, $\dot{n}_i$ is the molar rate of change of air in the tank, R is the gas constant, and T is the temperature inside the tank.

Provided that flow expressions define a volumetric flow across a valve at standard conditions, the ideal gas law can be utilized a second time to convert to a molar flow across a valve.

$$\dot{n} = \left(\frac{P_{atm}}{RT_{std}}\right) q \qquad (4)$$

where P_{atm} is the standard (atmospheric) pressure, T_{std} is the standard temperature, and again q is a volumetric flowrate. Thus

$$\dot{P}_i = \left(\frac{P_{atm}}{V_i}\right)\left(\frac{T}{T_{std}}\right)\left(\sum q_{in} - \sum q_{out}\right) \qquad (5)$$

Based on this general expression, a discrete-time model of the system can be developed. Using the switching conditions prescribed by the valve manufacturers, a least squares regression can be performed to identify model coefficients that represent parameters such as the valve coefficients, temperature influences, etc. For the simple case of modeling the pressure within a single tank, the results are presented in Figure 9. It can be seen that the hybrid model that considers both low-pressure drop and choke flow regimes is better able to capture the system dynamics than a model based solely on low-pressure drop flow.

Alternatively, mixed integer methods[28,29,30] can be used to develop strictly empirical hybrid descriptions of the process. Propositional logic can be used to formulate Mixed Integer Linear Programs (MILP) whose solution yields optimal coefficients and switching conditions for a variety of model forms including hybrid Volterra, autoregressive moving average (ARMA), and more general nonlinear state-space representations.

On a similar note, six process states can be considered in the modeling of the dynamics of the system. The pressure in each of the four tanks can act as states in the model, as well as two states that are not so obvious. The placement of the two supplemental valves leading into the two larger tanks causes some resistance to air flow, regardless of their position. This, in effect, makes the small sections of entrance tubing between the control valves and the supplemental valves act as two additional but very small tanks. The pressure in these two regions will act as the remaining process states. No pressure measurements are available in the areas, yet the size of these "tanks" and the nature of the system imply that the associated dynamics are extremely fast. A set of ordinary differential equations (ODEs) can be developed for the tank system to describe each respective state. Under the assumptions that these two extra tanks exhibit fast dynamics in comparison to the rest of the system, however, an approximation can be made that reduces the respective ODEs to algebraic relationships as the derivative term can be approximated as zero. This leads to the use of a system of differential algebraic equations (DAE) to describe the system, as well as motivating studies in the area.

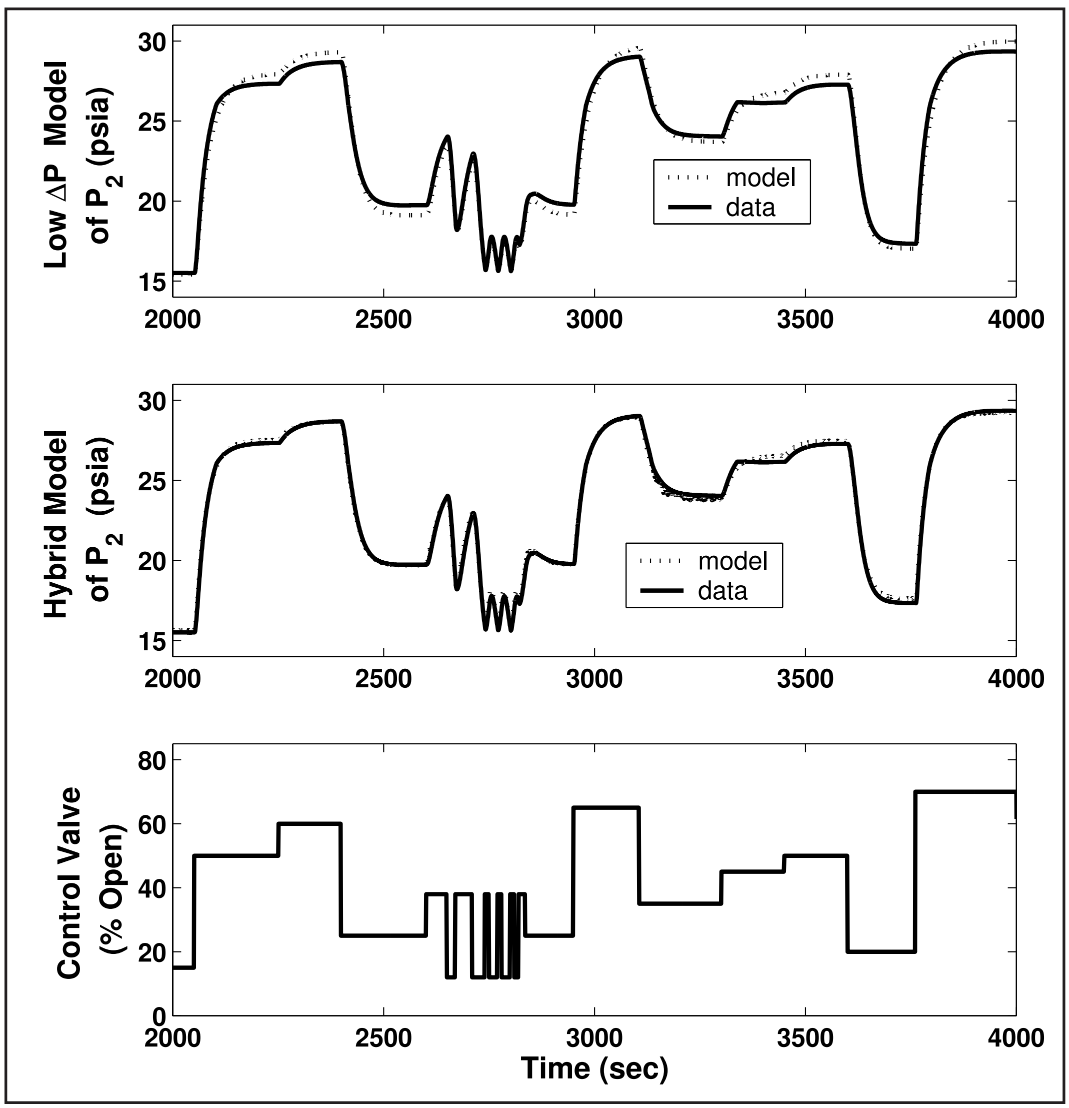

Figure 9. *Comparison of a fundamental low-pressure-drop flow model and a hybrid dynamic model in their ability to describe the pressure in a downstream tank.*

Additionally, the system has been utilized as a testbed for the development of advanced control strategies. In one case, the prioritized objective inferential control of unmeasured process states is considered. The system is operated in a 2-by-2 fashion with measurements of the downstream tank pressures available. The two upstream tank pressures are considered as the unmeasured process states to be controlled. Traditional Model Predictive Control (MPC) methods are often limited to the control of measured outputs and typically rely on a heuristic tuning to address the trade-off between satisfying different control objectives. A state-space modeling approach can be utilized to explicitly describe unmeasured process states. Using information from this state-explicit model and using propositional logic, a mixed-integer MPC algorithm[31] can be developed that relies on the online solution of an MILP or MIQP for the optimal control move. Such a formulation can allow for a more intuitive tuning in which control objectives, possibly involving unmeasured states, are met in order of their assigned priority.

CONCLUSIONS

Chemical process control education is often limited by the availability of practical "hands-on" educational tools. Few industrially relevant systems are available that offer both reasonable size and cost while providing interesting dynamics with the flexibility to be used in numerous contexts. This paper describes two such systems that provide students with the opportunity to actually apply and demonstrate experimentally many of the theoretical concepts that are fundamental to the subject. A small, experimental, two-tank system has been developed for use as a tool in process control education. The size and simplicity of the system lend themselves well to particular use in the undergraduate classroom. A similar yet more complex multivariable four-tank has also been developed. Its flexibility enables its use in a variety of applications. Many aspects of both the undergraduate and graduate-level process control curriculum can be presented. Additionally, the system is the focus of a variety of interesting research problems. Among these are studies on the hybrid dynamic nature of the flow through the system, and the systems' use as a testbed for advanced control schemes such as prioritized objective MPC.

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