Reprise: Solving Partial Differential Equations Using Excel 2000

by

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Introduction

The parabolic partial differential equation

$$\alpha * \partial^2 T/\partial x^2 = \partial T/\partial t \qquad (1)$$

with boundary conditions

At $t = 0$, $T = T_o$ at all x

At $t \geq 0$, $T = T_A$ at $x = 0$

and $\partial T/\partial x = 0$ at $x = L$

arises in the simulation of unsteady state heat conduction in a one-dimensional slab.

Initially the slab is at T_o. Then at $t \geq 0$, the surface ($x = 0$) is held at the constant temperature T_A. The other surface at $x = L$ is insulated.

In the above T is the temperature in K, t is the time in s and α is the thermal diffusivity in m^2/s. The parameters used are $T_o = 100$, $T_A = 0$ and $\alpha = 0.00002$.

The problem has been solved numerically by Cutlip and Shacham (1) using Polymath and the Method of Lines and by Taylor (2) using Maple and the Method of Lines. Subramanian and White (3) also used Maple to solve a similar problem but with a different boundary condition. It is the purpose of this study to compare the results of using Excel 2000 and finite differences with the results given in References (1) and (2).

Finite Difference Solution

The approach used here is that described by Rosen (4).

The temporal first derivative can be approximated by

$$\partial T/\partial t = (T_i^{k+1} - T_i^{k}) / \Delta t \qquad (2)$$

The second derivative can be approximated as (Crank-Nicolson method (5))

$$\partial^2 T/\partial x^2 = ½ *[(T_{i+1}^{k} - 2 T_i^{k} + T_{i-1}^{k})/ \Delta x^2 + (T_{I+1}^{k+1} -2 T_i^{k+1} T_{i-1}^{k+1})]/ \Delta x^2 \quad (3)$$

Letting

$$\Delta x = \Delta z * L \quad (4)$$

and then substituting Equations (2), (3) and (4) into Equation (1) and solving for T_i^{k+1} results in

$$T_i^{k+1} = (1/(\lambda+2)) * [\lambda T_i^{k} + T_{i+1}^{k} - 2 T_i^{k} + T_{i-1}^{k} + T_{i+1}^{k+1} + T_{i-1}^{k+1}] \quad (5)$$

where

$$\lambda = 2 \Delta z^2 L^2/ (\alpha \Delta t) \quad (6)$$

In the above, subscript i is the space index and superscript k is the time index.

Spreadsheet Implementation

In order for the computation to be stable, the value of λ must be (Chapra et. al. (5))

$$\lambda \geq 4 \quad (7)$$

If Δt is chosen as 50 s and $\Delta z = 0.05$ (with L = 1) then $\lambda = 5$. The spreadsheet can then be set up with 21 columns of z (z = 0, 0.05, 0.10…. 1.00) and with rows starting with t = 0 with increments of 50 s.

Each cell in the first row is set up to be equal to T_o. All other cells (except z = 0. and z = L) are set up to be equal to T_o initially and then to implement Equation (5). Note that each cell utilizes the cells on each side of itself as well as the cells in the row above it.

The spreadsheet iterates until the values in each cell change only by a very small amount. Use is made of an IF statement in each cell to be able to test an initialization parameter to see if the cell should be set to the initial value or to be calculated by Equation (5).

The cells at z = 0 in each row are set to T_A. The cells at z = L utilize the the first derivative form (Chapra et. al (5))

$$\partial T/\partial x = (3\, T_i - 4\, T_{i-1} + T_{i-2})/\, 2\, \Delta x \qquad (8)$$

Since this must be zero, the value of T in the cells at z = L (at all times) becomes

$$T_i^{\,k+1} = (4\, T_{i-1}^{\,k+1} - T_{i-2}^{\,k+1})/\ 3 \qquad (9)$$

Results and Conclusions

Figure 1 is an excerpt from the final spreadsheet (after all the iterations were completed). The values of the temperature at time = 6000 s (not shown in Figure1) are shown in Table 1 and compared with the results in References (1) and (2).

z	Reference 1 $\Delta z = 0.05$	Reference 2 $\Delta z = 0.10$	This Work $\Delta z = 0.05$
0.	0.	0.	0
0.2	31.68	31.71	31.677
0.4	58.47	58.49	58.473
0.6	77.49	77.46	77.489
0.8	88.29	88.22	88.287
1.0	91.72	91.66	91.716

Table 1 Comparison of Temperatures at 6000s

The results compare well but it should be noted that Taylor used a space increment of 0.1 while Cutlip and Shacham and this work used 0.05.

Shacham and Cutlip (6) compare the use of different numerical programming packages (7) on a particular problem . In general, it appears that the selection of a particular package depends on the problem being solved, the familiarity of the user with that package, the particular package's strength and any cost that might be involved. (The author was quoted $1695 for Maple 6.1 as a standalone system). For this problem the use of Excel was straight-forward, did not require any training beyond the use of routine Excel (no VBA required) and involved no extra costs.

The complete spreadsheet may be downloaded from the author's website: http://ourworld.cs.com/edwardmemrose

References

1. Cutlip, M. B. and M. Shacham, “The Numerical Method of Lines for Partial Differential Equations”, *CACHE News*, Fall 1998, p 18

2. Taylor, R., “Engineering Computing with Maple: Solution of PDEs via the Method of Lines”, *CACHE News*, No. 49 Fall 1999 p 5

3. Subramanian, V. R. and R. E. White, “Solving Differential Equations With Maple”, *Chemical Engineering Education*, Vol 34 No 3 Fall 2000, p 328

4. Rosen, E. M. , “Excel 7.0: Partial Differential Equations”, *CACHE News* No. 48, Spring 1999.

5. Chapra, S. C. and R. P. Canale, *Numerical Methods for Engineers*, McGraw-Hill, New York (1988)

6. Shacham, M. and M. B. Cutlip, “A Comparison of Six Numerical Software Packages for Educational Use in the Chemical Engineering Curriculum”, Paper 2520 from American Society of Engineering Education Annual Conference, Seattle, Wash. June 28-July 1, 1998

7. Phillips, J. E. and J. E. DeCicco, “Choose the Right Mathematical Software”, *Chemical Engineering Progress*, July 1999, p 69.

Heat Conduction In a One Dimensional Slab
Unsteady State- CACHE News Fall 1998

Initial	0		
Alpha	0.00002	Lambda	5
To	100		
TA	0		
L	1		
Del z	0.05		
Del t	50		

Index	Time (sec)	0	0.05	0.1	0.15	0.2	0.25	0.3	0.35
1	0	0	100	100	100	100	100	100	100
2	50	0	70.82039	95.74275	99.37888	99.90938	99.98678	99.99807	99.99972
3	100	0	56.50161	87.30734	97.22225	99.45964	99.90145	99.98275	99.99706
4	150	0	48.06492	79.64224	93.78491	98.41842	99.64142	99.92475	99.98505
5	200	0	42.44938	73.30869	89.93489	96.8201	99.12423	99.7821	99.94974
6	250	0	38.39662	68.11952	86.1297	94.85494	98.33545	99.51831	99.87246
7	300	0	35.30428	63.82057	82.55485	92.69978	97.31362	99.11596	99.73527
8	350	0	32.84837	60.20517	79.26696	90.47868	96.11603	98.57693	99.52568

Figure 1 Excerpt from Final Spreadsheet