TEACHING PROCESS CONTROL WITH A NUMERICAL APPROACH BASED ON SPREADSHEETS

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The traditional method for teaching process control courses uses analytic techniques based on Laplace transforms to solve the relevant differential equations.[1-9] The mathematical manipulations involved in these analytic solutions are so complex and non-intuitive, however, that students can lose sight of the physical significance of the results. Numerical solutions offer a remedy to this problem and can be used in conjunction with traditional analytic solutions to strengthen the instruction of process control. We emphasize that numerical solutions are *not* intended to replace analytic methods, but should instead be used in addition to analytic methods.

The use of computers in obtaining numerical solutions can give an enhanced physical intuition and understanding that can be difficult to achieve from analytic solutions alone. As a report in *Science* claims, "Many physics students ... can solve the calculus-based equations at the heart of many laws of nature, but they lack an intuitive feel for how they work.[10] In contrast, numerical solutions solve the fundamental equations directly, allowing students to focus on the physical problem rather than on mathematical manipulations and approximations.[11] The interactive nature of computers allows "what-if" experiments in which values of parameters are changed, and the results are dislayed immediately in graphical form. The usefulness of this approach is summarized by the

	A	B	C	D	E
1	Process Variables		Disturbance		
2	K =	5	t_{step} =	1	
3	τ =	2	for t<t_{step}	0	
4	ζ =	1	for t≥t_{step}	1	
5					
6		Time Step	Initial Values		
7	Δt =	0.01	y(0) =	0	
8			y'(0) =	0	
9					
10	t	f	y	y'	y"
11	0	if(A11<D$2,D$3,D$4)	D7	D8	(B$2*B11-2*B$4*B$3*D11-C11)/(B$3)^2
12	A11+B$7		C11+D11*B$7	D11+E11*B$7	
13					
14					
15					
16					

Figure 1. *Spreadsheet used to determine the response of a 2nd order process to a step change in the disturbance. The step function is implemented with an IF function of the form IF (expression, value if true, value if false). Arrows indicate that cells should be copied and pasted downward for approximately 5,000 to 10,000 rows.*

title of a recent article in *Chemical and Engineering News*: "Thinking Instead of Cookbooking: When Computers Take Over the Dirty Work ... Students Can Focus on the Bigger Picture."[12]

The differential equations that arise in process control applications are readily solved numerically by using simple spreadsheets that can be constructed by the students in less than five minutes. Students can experiment with different control schemes and parameters in order to gain an understanding of how each parameter affects the response of the system. They develop an intuitive feel for how a system will respond to input changes and how this response can be controlled. Then, they discover how to optimize the control.

This strategy has been used in the process control course at Tulane. The numerical approach is used first to introduce a topic, allowing students to obtain a good physical understanding before proceeding. The topic is then addressed more fully with the traditional analytical approach based on Laplace transforms. Students follow the analytical approach more easily at this point since they already have a solid physical understanding from the numerical approach.

DESCRIPTION OF APPROACH

This section describes how the numerical approach using spreadsheets can be used to teach most major topics in a process control course, including process dynamics, frequency response analysis, feedback control, and advanced control
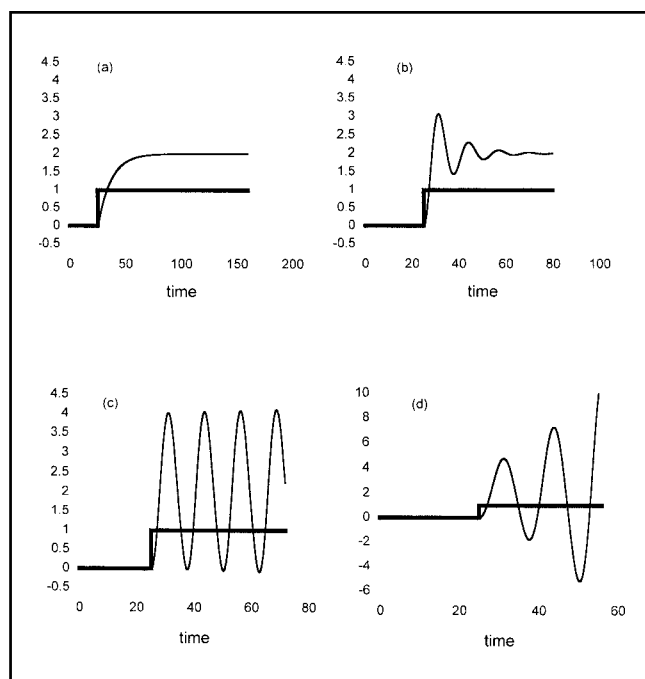

Figure 2. Response of a 2nd order process to a step change in the disturbance for (a) $\zeta = 3$ (b) $\zeta = 0.2$ (c) $\zeta = 0$ (d) $\zeta = -0.1$ The bold line is the disturbance, and the thin line is the response.

techniques such as feedforward and cascade control.

Process Dynamics

As an example, the response of a linear second-order process is examined.[1-9] A linear second-order process is described in general by

$$\tau^2 y'' + 2\zeta\tau y' + y = Kf(t) \tag{1}$$

where y is the response of the process (output), $y' = dy/dt$, $y'' = d^2y/dt^2$, f is the disturbance (input), K is the gain, τ is the characteristic time, and ζ is the damping factor.

Differential equations can be solved numerically using Euler's Method. This method is implemented for second-order differential equation by repeatedly applying the following algebraic equations for small time increments, Δt:

$$y(t + \Delta t) = y(t) + y'(t)\Delta t \tag{2}$$

$$y'(t + \Delta t) = y'(t) + y''(t)\Delta t \tag{3}$$

Note that the initial values of y and y' must be specified, and the values of y''(t) are obtained by rearranging Eq. (1).

$$y''(t) = \frac{Kf(t) - 2\zeta\tau y'(t) - y(t)}{\tau^2} \tag{1a}$$

Below, we present the implementation of this method for a step change in f(t).

The spreadsheet used to solve this problem is shown in Figure 1. The results are easily displayed in graphical form by plotting y and f together as functions of time. All parameters are defined at the top of the spreadsheet, and their cell locations are referenced in the relevant equations. Upon changing parameter values, the graphical display of the results is updated immediately, without rewriting any of the spreadsheet.

The physical significance of the damping factor, ζ, in a second-order linear differential equation can be demonstrated with this approach by comparing the response to a step change for different values of ζ. For $\zeta > 1$, the response is overdamped, and it reaches a steady state without oscillating (Figure 2a). For $0 < \zeta < 1$, the response is underdamped, and it exhibits decreasing oscillations as it reaches a steady state (Figure 2b). For $\zeta = 0$, the response is undamped, and it oscillates indefinitely (Figure 2c shows a slight increase in amplitude with time, due to numerical error—see Discussion section). For $\zeta < 0$, the response is unstable, and it increases without bound (Figure 2d). All of these results are generated and graphically displayed in a matter of seconds once the spreadsheet is constructed.

Frequency Response Analysis

The frequency-dependent response to an oscillating disturbance is important in many fields, including process control. The traditional method of teaching frequency response analysis is given in process control textbooks.[1-9] A second-order process (Eq. 1) is examined here, and the spreadsheet used to solve this problem (Figure 3) is just a slight modification of

the spreadsheet used for the step function input (only the disturbance is different).

The frequency response of the system can be addressed by comparing the response obtained with different values of the angular frequency, ω. When the frequency is small, the system has sufficient time to react to the changing disturbance, and the response is nearly in phase with the disturbance (Figure 4a). When the frequency is increased, however, the system does not have sufficient time to react, and the response increasingly lags behind the disturbance (Figures 4b and 4c). Additionally, the amplitude of the response usually decreases with increasing frequency (Figures 4a, 4b, and 4c). For $\zeta < 1$ and small frequencies, however, the behavior of a linear second-order system is unusual in that the amplitude increases with increasing frequency (Figure 4d). Note that the immediate graphical results allow students to quickly and easily experiment with different values of ω and ζ.

	A	B	C	D	E
1	Process Variables		Disturbance		
2	K =	1	A =	1	
3	τ =	2	ω =	0.01	
4	ζ =	1.5			
5					
6	Time Step		Initial Values		
7	Δt =	0.5	y(0) =	0	
8			y'(0) =	0	
9					
10	t	f	y	y'	y"
11	0	D$2*sin(D$3*A11)	D7	D8	(B$2*B11-2*B$4*B$3*D11-C11)/(B$3)^2
12	A11+B$7		C11+D11*B$7	D11+E11*B$7	
13					
14					
15					
16					

Figure 3. *Spreadsheet used to determine the response of a 2nd order process to an oscillating disturbance. Arrows indicate that cells should be copied and pasted downward for approximately 5,000 to 10,000 rows.*
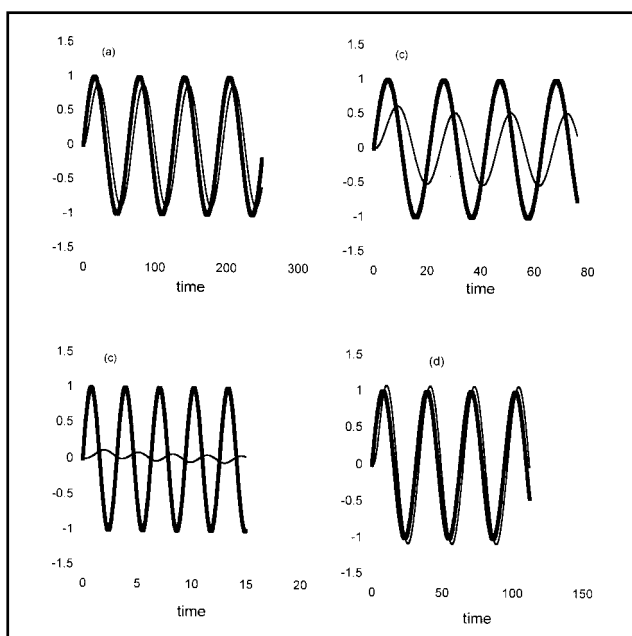
Feedback Control

A feedback control mechanism measures the output of the process, compares it to the desired value (the set point), and then alters an input to the process in order to bring the output closer to the desired value.[1-9]

The output of a proportional-integral-derivative (PID) controller is given by

$$y_c = K_c \varepsilon + \frac{K_c}{\tau_1} \int_0^t \varepsilon dt + K_c \tau_D \frac{d\varepsilon}{dt} \qquad (4)$$

where $\varepsilon = y_{sp} - y$, y_{sp} is the set point, and y is the output of the process. When the system is not under any control, the values of K_c and τ_D are set equal to zero, while τ_1 is set equal to infinity. The integral term can be calculated numerically as

$$\int_0^t \varepsilon dt = \sum_i \varepsilon(t_i)\Delta t \qquad (5)$$

and the derivative term can be calculated numerically as

$$\frac{d\varepsilon(t)}{dt} = \frac{\varepsilon(t) - \varepsilon(t - \Delta t)}{\Delta t} \qquad (6)$$

The numerical approach is applied here to the feedback control of a process consisting of three first-order systems in series. The dynamics of the other parts of the control loop (e.g., measuring device) are not included for simplicity, but can easily be included if desired (as pointed out in the Discussion section). A process consisting of three first-order systems in

Figure 4. *Response of a 2nd order process to an oscillating disturbance for (a)* $\zeta = 1.5$, $\omega = 0.1$; *(b)* $\zeta = 1.5$, $\omega = 0.3$; *(c)* $\zeta = 1.5$, $\omega = 2$; *(d)* $\zeta = 0.5$, $\omega = 0.2$. *The bold line is the disturbance, and the thin line is the response.*

series is described by three coupled first-order differential equations,

$$\tau_i y_i' + y_i = K_i f + K_p y_c \qquad i = 1 \qquad (7)$$

$$\tau_i y_i' + y_i = K_i y_{i-1} \qquad i = 2, 3 \qquad (8)$$

where i is the system number. These coupled differential equations are numerically integrated using Euler's method by repeatedly applying the algebraic equations

$$y_i(t + \Delta t) = y_i(t) + y_i'(t)\Delta t \qquad i = 1, 2, 3 \qquad (9)$$

where the $y_i'(t)$ are obtained from Equations 7 and 8. The spreadsheet used to solve this problem is shown in Figure 5.

By experimenting with different values of the control parameters (K_c, τ_1 and τ_D), the relationship between each control parameter and the response can be determined. If proportional-only control is used (*i.e.*, $\tau_D = 0$ and τ_1 = a large number that approximates ∞), the response is offset from the set point (Figure 6a). Increasing the value of K_c will minimize this offset (Figure 6b), but the system can become unstable if K_c is too large (Figure 6c). Adding integral control (*i.e.*, decreasing τ_1 from ∞) will eliminate this offset (Figure 6d). But if the value of τ_1 is too small, the system becomes unstable (Figure 6e). Adding derivative control (*i.e.*, increasing τ_D from 0) stabilizes the system (Figure 6f). This stabilization allows a larger K_c and a smaller τ_1 to be used, but a large τ_D value also slows the response.

The values of the control parameters should be chosen such that a quick response with small oscillations and no offset is achieved. The Zeigler-Nichols tuning method is one way to obtain advantageous values for the three control parameters, in which

$$K_c = \frac{K_c^{max}}{1.7} \qquad (10a)$$

$$\tau_1 = \frac{P_u}{2} \qquad (10b)$$

$$\tau_D = \frac{P_u}{8} \qquad (10c)$$

where K_c^{max} is the maximum value of K_c for which the response is stable with a proportional-only controller, and P_u is the period of oscillation of the response at K_c^{max}. The value of K_c^{max} is found by trial
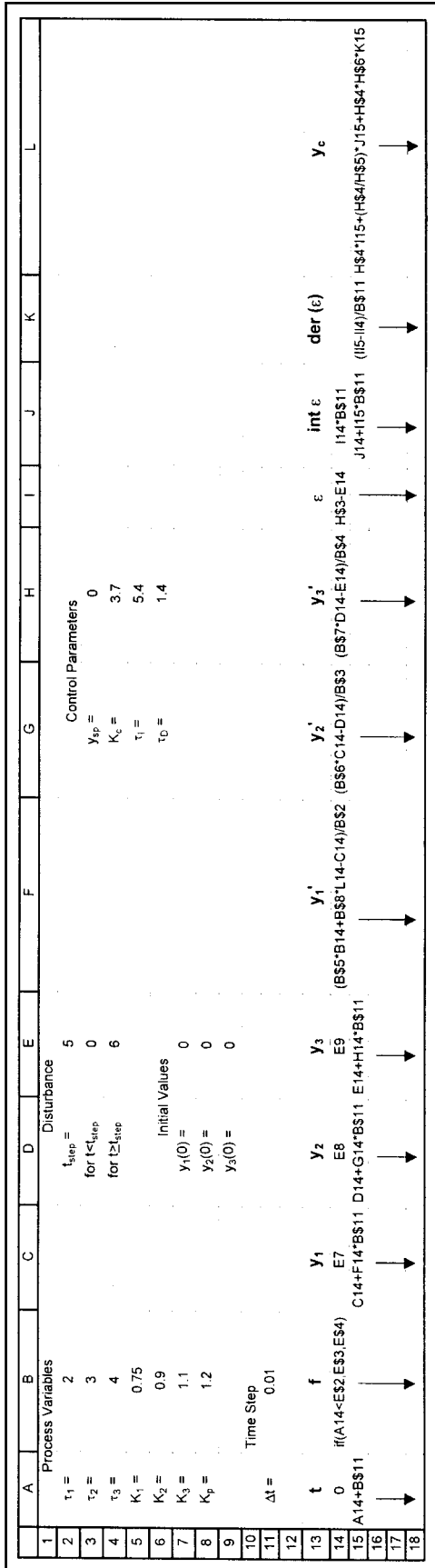
Figure 5. Spreadsheet used to determine the response of a process consisting of three 1st order systems in series with feedback control (PID controller). Arrows show that cells should be copied and pasted downward for 5,000-10,000 rows.

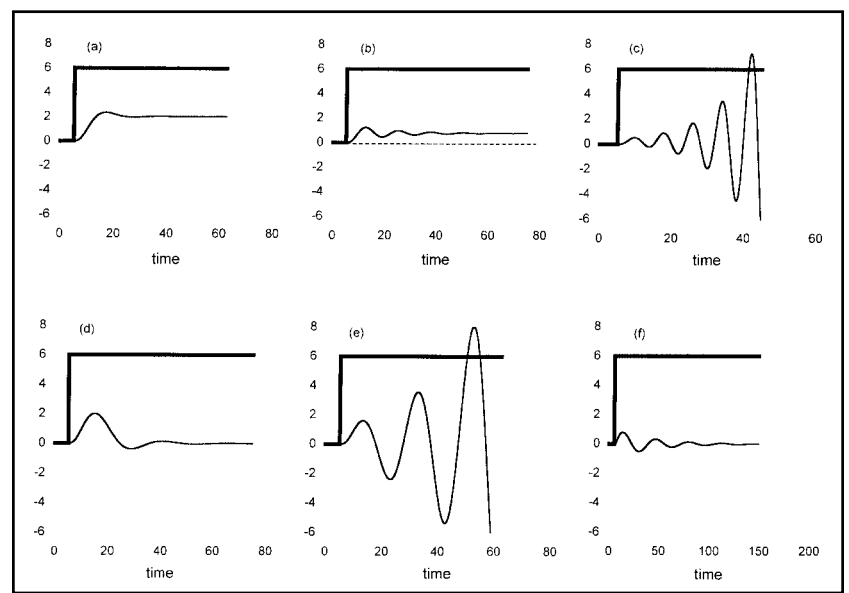|	A	B	C	D	E	F	G	H	I	J	K	L
1	Process Variables			Disturbance			Control Parameters					
2	τ_1 =	2		t_{step} =	5		y_{sp} =	0				
3	τ_2 =	3		for t<t_{step}	0		K_c =	3.7				
4	τ_3 =	4		for t≥t_{step}	6		τ_1 =	5.4				
5	K_1 =	0.75					τ_D =	1.4				
6	K_2 =	0.9		Initial Values								
7	K_3 =	1.1		$y_1(0)$ =	0							
8	K_p =	1.2		$y_2(0)$ =	0							
9				$y_3(0)$ =	0							
10	Time Step											
11	Δt =	0.01										
12												
13	t		y_1	y_2	y_3	y_1'	y_2'	y_3'	ε	int ε	der (ε)	y_c
14	0	if(A14<E$2,E$3,E$4)	E7	E8	E9	(B$5*B14+B$8*L14-C14)/B$2	(B$6*C14-D14)/B$3	(B$7*D14-E14)/B$4	H$3-E14	I14*B$11	(I15-I14)/B$11	
15	A14+B$11		C14+F14*B$11	D14+G14*B$11	E14+H14*B$11					J14+I15*B$11		H$4*I15+(H$4/H$5)*J15+H$4*H$6*K15

Figure 6. *Response of a process consisting of three first-order systems in series with feedback control to a step change in the disturbance. (a) P-only, $K_c = 1$; (b) P-only, $K_c = 4$; (c) P-only, $K_c = 15$; (d) PI: $K_c = 1$, $\tau_1 = 5$; (e) PI: $K_c = 1$, $\tau_1 = 1.3$, (f) PID: $K_c = 1$, $\tau_1 = 1.3$, $\tau_D = 15$. The bold line is the disturbance, and the thin line is the response.*
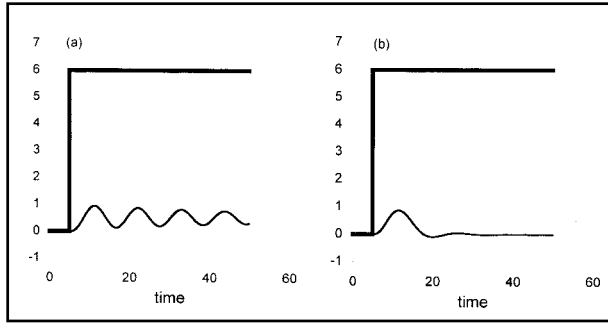
Figure 7. *Tuning of PID parameters with Ziegler-Nichols method, for a process consisting of three first-order systems in series with feedback control. (a) Determination of* K_c^{max} *and* P_u; *(b) PID with Ziegler-Nichols parameters:* $K_c = 3.7$, $\tau_I = 5.4$, $\tau_D = 1.4$. *The bold line is the disturbance, and the thin line is the response.*

and error to be 6.3 (Figure 7a), and the value of P_u is observed to be 10.8. The response using the Ziegler-Nichols parameters is shown in Figure 7b.

Feedforward Control

A feedforward control mechanism measures the disturbance and uses this measured value to adjust an input variable with the goal of keeping the process output at the desired value.[1] The output of a simple feedforward controller is given by

$$y_c = Ay_{sp} - Bf \qquad (11)$$

where A and B are controller parameters that will depend on the particular process to be controlled.

The numerical approach is applied here to the feedforward control of a process consisting of three first-order systems in series (Eq. 7 and 8). The spreadsheet for this problem is shown in Figure 8. Perfect control can be obtained by choosing the parameters such that the system is at steady state with the process output at the set point (i.e., $y_1' = y_2' = y_3' = 0$ and $y_3 = y_{sp}$). From equations 7 and 8, it is easily found that the parameter values that yield perfect control are $A = 1/(K_p K_2 K_3)$ and
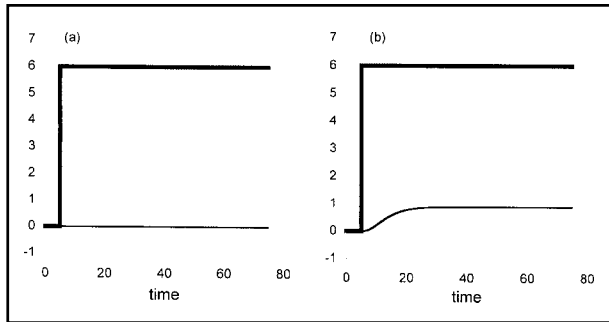

Figure 9. *Response of a process consisting of three first-order systems in series with feedforward control to a step change in the disturbance. (a)* $A = 1/(K_p K_2 K_3) = 0.842$ *and* $B = K_1/K_p = 0.625$; *(b)* $A = 0.842$ *and* $B = 0.5$. *The bold line is the disturbance; the thin line is the response.*
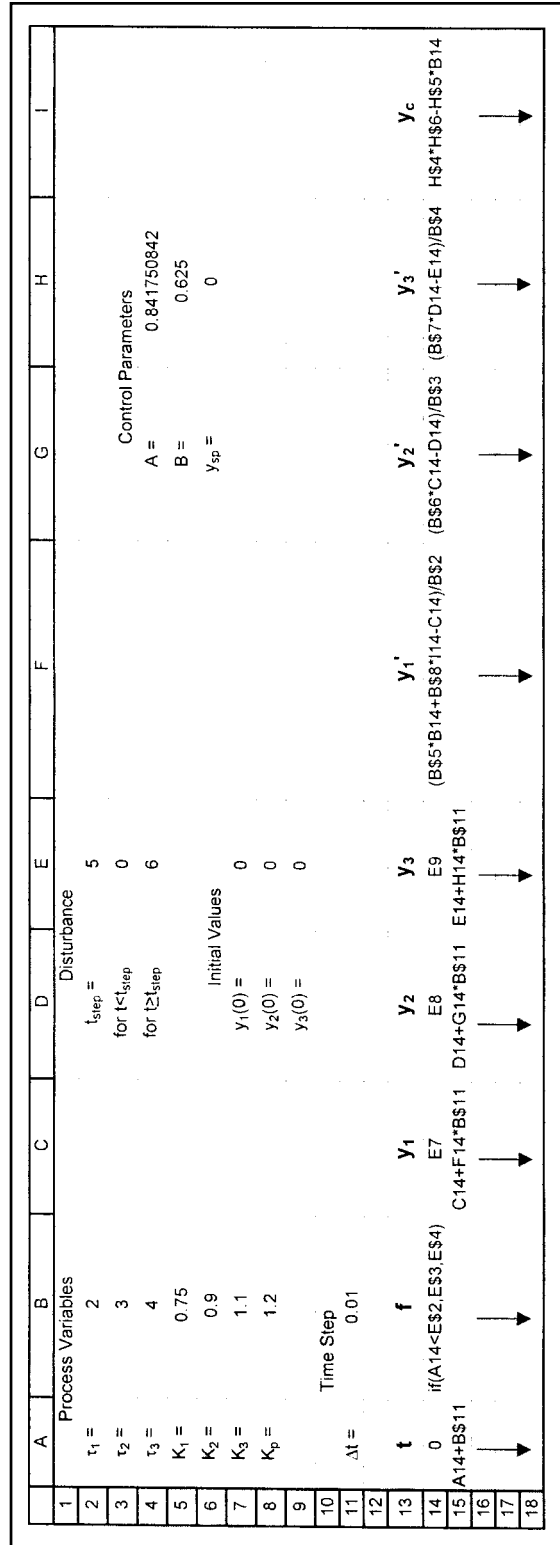

Figure 8. Spreadsheet used to determine the response of a process consisting of three first-order systems in series with feedforward control. Arrows indicate that cells should be copied and pasted downward for approximately 5,000 to 10,000 rows.

Figure 10. Spreadsheet used to determine the response of a process consisting of three first-order systems in series with cascade control. Arrows indicate that cells should be copied and pasted downward for 5,000 to 10,000 rows.
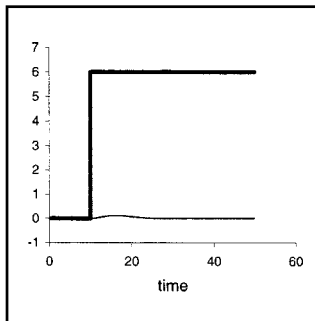
Spreadsheet contents (Figure 10):

	A	B
Process Variables		
τ_1 =	2	
τ_2 =	3	
τ_3 =	4	
K_1 =	0.75	
K_2 =	0.9	
K_3 =	1.1	
K_p =	1.2	

Disturbance		E
t_{step} =		10
for $t<t_{step}$		0
for $t\geq t_{step}$		6

Control Parameters		H
K_{cp} =		2
τ_{ip} =		5
y_{sp} =		0
K_{cs} =		10

Initial Values		E
$y_1(0)$ =		0
$y_2(0)$ =		0
$y_3(0)$ =		0

Time Step		
Δt =	0.01	

A (t)	B (f)	C (y_1)	D (y_2)	E (y_3)	F (y_1')	G (y_2')	H (y_3')	I (ε_p)	J (int(ε))	K (y_{cp})	L (ε_s)	M (y_{cs})
0	if(A14<E\$2,E\$3,E\$4)											
A14+B\$11	if(A14<E\$2,E\$3,E\$4)	C14+F14*B\$11	D14+G14*B\$11	E14+H14*B\$11	(B\$5*B14+B\$8*M14-C14)/B\$2	(B\$6*C14-D14)/B\$3	(B\$7*D14-E14)/B\$4	H\$5-E14	I14*B\$11 / J14+I15*B\$11	H\$3*I14+(H\$3/H\$4)*J14	K14-C14	H\$6*L14

$B = K_1 / K_p$. As shown in Figure 9a, perfect control is indeed achieved with these parameters. Perfect control is no longer achieved when $A \neq 1/(K_p K_2 K_3)$ or $B \neq K_1 / K_p$ (as shown in Figure 9b). Since real processes are generally not simple with accurately known parameters, perfect control is only idealistic, not practical.

Cascade Control

Cascade control uses two control loops (primary and secondary).[1] The primary control compares the process output to the desired value (set point), yielding a second set point to be used for a secondary control. The secondary control compares an intermediate quantity to this second set point to determine how to alter an input variable.

The example of a process consisting of three first-order systems in series (Eq. 7 and 8) is used to examine cascade control. The intermediate quantity used in the secondary control loop is the output of the first-order process (y_1). A proportional-integral controller is used for the primary controller, and a proportional-only controller is used for the secondary controller. The spreadsheet used to solve this problem is shown in Figure 10.

The response of the system with cascade control is shown in Figure 11 - this response is superior to the response with feedback control (Figure 7b). (Note that this example is somewhat artificial in that the secondary control loop consists of only a first-order process and will be stable for any value of the secondary controller gain. Therefore, an arbitrarily large value of the secondary controller gain can be used to make the response arbitrarily fast. This arbitrarily fast response is not possible in general, *e.g.*, if the secondary loop includes dead time or a process higher than second-order).

DISCUSSION

Implementation of Approach

This numerical approach using spreadsheets was implemented in the process control course at Tulane as follows: first, a topic is introduced in a lecture, and the governing equations are derived; next, the class moves on to our computer lab, where students solve the governing equations numerically (all students do this individually on separate computers), and the physical significance of the results is discussed; finally, the traditional analytic solutions based on Laplace transforms are taught, in lecture format.

Homework assignments include problems requiring numerical solutions using spreadsheets, problems requiring analytical solutions, and problems that use the Control Station software package.[13] Some problems require that students compare results from numerical solutions to results from analytical solutions. For example, one

Figure 11. *Response of a process consisting of three first-order systems in series with cascade control to a step change in the disturbance (primary controller: K_c=2 and τ_1=5, secondary controller: K_c=10). The bold line is the disturbance, and the thin line is the response.*

homework problem requires that students find the maximum value of a controller gain for a proportional-only controller in a certain process by three methods: by trial and error with numerical solutions, by deriving the transfer function and finding the gain that leads to positive real parts of its poles, and by the Bode stability criterion using analytical expressions for phase lags and amplitude ratios. The students compare the results for the maximum controller gain from these different methods and find them to be the same (within numerical error).

The exams test the students' knowledge of applying numerical methods to process control problems, in addition to the traditional process control material. One of the exams includes a computer part (given in class in our computer computer lab), where students solve a problem numerically with a spreadsheet and turn in the printed result. The other exams have problems in which students must show how to set up a spreadsheet to numerically solve a given problem, providing all of the relevant equations.

Students found the numerical approach using spreadsheets to be extremely useful in understanding the concepts underlying process control. In unsolicited comments on the course evaluations, two-thirds of the students remarked that the numerical approach was the most valuable aspect of the course.

The students also seemed to genuinely enjoy this approach. When problems were solved with this method in the computer lab, students were often so eager to discover the effects of changing some parameters that they would proceed ahead of the discussion. They would also occasionally continue experimenting with the effects of different parameters after the class had ended.

Other Issues

The numerical approach is more general than the analytic approach, in that it can also be applied to nonlinear differential equations, *i.e.*, a linearization approximation is not necessary as it is for the analytic approach based on Laplace transforms. To emphasize this point, a homework problem was given in which students investigate the frequency response for a process described by the nonlinear differential equation $y' + y^a = f$ (where a is the number of letters in their last name divided by five), and then use the results to construct Bode and Nyquist diagrams.

A concern with the numerical approach, of course, is that there is numerical error in the results. Students should be aware of the numerical error and that the error can be reduced by decreasing the time step Δt or by using a more sophisticated integration method (*e.g.*, Runge-Kutta or a predictor-corrector method). A reasonable time step for these problems is $\Delta t = \tau / 100$, where τ is the smallest characteristic time for the system.

Although excluded here for simplicity, it is straightforward to include in this approach the dynamics of other elements of the control loop, such as actuators (*e.g.*, valves) and measuring devices. Including the dynamics of these elements would amount to including a few more coupled differential equations, which translates to a few more columns on the spreadsheet.

Dead time is also straightforward to include in this approach. To introduce dead time to a variable y, a new variable, y_{+dead}, is defined such that $y_{+dead}(t) = y(t - t_{dead})$. The values for y_{+dead} are obtained in the spreadsheet from the values of y, by setting the cell for y_{+dead} at the time, t, equal to the value of the cell for y at the time $t - t_{dead}$ (*i.e.*, $t_{dead} / \Delta t$ rows above in the spreadsheet).

The present approach is different than, but complementary to, an approach that uses packaged software (such as Control Station[13]) for teaching process control. In the present approach, students are in fact solving the governing equations themselves, with a numerical method rather than an analytical method. In contrast, the Control Station software[13] presents results without requiring that students solve the equations.

CONCLUSION

In the usual method for teaching process control, students are taught to solve the relevant differential equations analytically by using Laplace transforms. This method involves complex mathematical manipulations, which can cause students to lose sight of the physical significance of the problem. The main goal of a process control course should be to provide a general understanding and intuitive feel for how physical processes behave and how they can be controlled. Numerical solutions for process control problems are extremely easy to obtain using spreadsheets created by students themselves. This approach allows students to concentrate on what is physically happening as opposed to the complex mathematics, yet the students solve the problems themselves (*i.e.*, the solution is not given to them by packaged software). This approach has been used in the Process Control course at Tulane, and student feedback has been extremely positive.

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