A FEMLAB Study:
The Effect of Heat Transfer on Flow Field At Low Reynolds Numbers In Vertical Tubes

by

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Introduction

The effect of heating or cooling a fluid at low Reynolds number flowing upward in a vertical tube has been studied by Hanratty, Rosen and Kabel (1) and by Scheele, Rosen and Hanratty (2). Rosen (3) used series representations of the velocity and temperature fields to present an approximate solution to the equations of motion and energy. The results were reported by Rosen and Hanratty (4).

In this study a numerical solution to the problem using finite elements is made using FEMLAB/Matlab (5, 6). The study is limited to the prediction of the point of flow inversion (null point) at the center of the tube during heating.

Description of the Flow Field

Experiments (3) were carried out using water in a long vertical glass tube inside an outer glass tube in which heated water was circulated. A dye was used to observe the flow field. Water in laminar flow (parabolic velocity profile) entered the heated section.

If the dye flooded the field and then was allowed to be swept out with the clear entering flowing water, a long cigar shaped area of dye appeared in the center of the of the tube. Its beginning location (the inversion or null point) was a function of the flow rate of the water and the temperature difference between the entering water and the temperature of the heated portion of the tube. Figure (1) indicates the coordinate system and nomenclature used in the study.

At the start of the heated section the water at the inner tube wall is accelerated more than the water in the center of the tube. As a result the velocity profile begins to flatten out. If the temperature difference between the wall and fluid is large enough, the center line velocity continues to fall until there is a flow reversal. Farther downstream instabilities may cause the flow field to become turbulent.

It is the location downstream at which the center line velocity goes to zero that is the object of this study. It is desired to compare the FEMLAB solution to experiment.

Equations Describing the Problem

FEMLAB (7) generally states the momentum (Navier-Stokes) and Energy Equations in in a vector form (steady state):

***Navier-Stokes*:**

$$-\nabla\cdot h\,(\nabla \mathbf{u} + (\nabla \mathbf{u})^{T}) + r\,(\mathbf{u}\cdot\nabla)\,\mathbf{u} + \nabla p = \mathbf{F}$$

The relationship of this equation to a 2D cartesian coordinate system is shown in Figure 2 (8). Figure 3 is a cylindrical coordinate system formulation (9).

The body force in the z direction (vertical) is a gravity term and is set to (the Boussinesq relationship):

Fz = alpha* g*rho0*(T-To)

where

alpha = coefficient of expansion
g = acceleration due to gravity
rho0 = density at the inlet conditions
T = fluid temperature
To = fluid temperature at inlet

***Energy Equation*:**

$$\nabla\cdot(-k\,\nabla T) = Q - r\,\mathrm{Cp}\,\mathbf{u}\,\nabla T$$

This is the non-conservative form of the energy equation and implies that $\nabla \mathbf{u} = 0$ and is appropriate for incompressible fluids.

Boundary Conditions

Four boundary conditions were set up:

1. The flow field is considered to be symmetric around the center line. Therefore there are no velocity components perpendicular to the center line.
2. The tube exit is considered to be at zero pressure.
3. The entrance velocity has a parabolic profile and the fluid (water) is at To.
4. Two types of boundary conditions were considered at the tube wall.

a. Constant Temperature set equal to Td. This neglects the resistance of the tube wall.

b. Specified flux. At the tube wall the the flux is

   kc*($\partial T / \partial r)_{r=a}$ = ks * (Td-T)/(a ln (d/a))

   where

   kc = thermal conductivity of fluid (water)
   T = Temperature of fluid
   r = radial distance
   ks = thermal conductivity of tube (Pyrex)
   Td = Temperature of the outer tube wall
   a = inside tube radius
   d = outside tube radius

Physical Properties

The physical properties of water are taken from (10). The properties are evaluated at the entrance temperature, To, by linear interpretation about the closest values found in the reference's tables. The data and calculations, given in Figures 4A and 4B generally reproduced those listed in (3). The coefficient of expansion (alpha) and temperature coefficient of viscosity (b) are taken from (3). Psi is calculated as a consistency check with (3).

Setting up FEMLAB

The general model to be used is first specified in the Model Navigator – new File:

Chemical Engineering Module
- *Axisymmetry*
  - *Momentum Balance*
    - *Navier-Stokes*
      - *Stationary*

Various choices are then made from the pull down menu:

Draw

The geometry of the long tube is represented as a long rectangle from the center to the inside radius of the tube.

An arbitrary rectangle is drawn (from the rectangle icon):

The rectangle is double clicked and a window appears. The length Y (z direction) is arbitrary but long enough to reflect the flow reversal point noted in the experiments.

The following values are entered in the window (In order to follow experiments more easily, actual dimensions are used rather than putting the equations in dimensionless form):

Xmin = 0. Xmax = 0.01095 m (inside tube radius)
Ymin = 0. Ymax = 1.0 m (this varies depending on the experiment)

The “zoom extents” button is clicked to center the long tube.

Options

Add/Edit Constants (Data for Run 24)

The physical properties of water for this run are given in Figures 4A and 4B. The “set” and “apply” buttons are clicked each time a value is entered. The Nomenclature indicates the general symbols used by FEMLAB and the symbols used in the simulation

Tin (same as To)	300.983	Kelvins
Cp	4178.72	J/(kg – K)
kc	0.61898	J/(s-m-K)
g	9.814	m/s^2
To	300.983	Kelvins
Uin (same as v_{max})	.0252601	m/s
alpha	0.000283914	1/Kelvins
rho0	996.309	kg/m^3
muo	0.00083908	kg/(m-s)
b	0.0218595	1/Kelvins)
Td	315.0944	Kelvins
fact	100.	This is a parameter that was varied to 50, 20, 10…1 in a series of runs to force convergence.

Subdomain

The subdomain generally refers to the equations being solved. There is a subdomain for the Navier-Stokes equations and one for the Energy equation.

Subdomain Settings (Navier-Stokes)

Coefficients (FEMLAB symbol and simulation symbol used)

? = rho0 Density
? = mu Dynamic Viscosity
Fr = 0 Volume force r direction
Fz = FY Volume force z direction

Options

This accounts for the buoyancy force variation and the variation of viscosity with temperature. The values of ko and Cpo are constants:

Add/Edit Expressions
Variable Name = FY
Add
Definition
*alpha*g*rho0*(T-To)/fact*

Variable Name = mu
Add
Definition
muo/(1+b(T-To))*

Boundary Settings

1	LHS = slip	(Left Hand Side - center line symmetry)
2	v = Uin*(1-s^2)	(s is a parameter going from 0 to 1 at input))
3	p = 0	(exit pressure is zero)
4	RHS = no slip	(Right Hand Side - at the wall)

Multiphysics

Add/Edit Modes
Chem Axi, Convection-Conduction (click on >>)
Non-Consevative (submode)

Subdomain Settings (Convective-Conduction) The general FEMLAB sysmbol is equated to the simulation symbol:

? = rho0 Density
Cp = Cp Heat Capacity
k = kc Thermal Conductivity (Isotropic)
Q = 0 Heat source
u = u Radial Velocity
v = v z direction velocity

Init
$T(t_o) = To$

Boundary Settings (Energy Equation)

1 LHS = insulation
2 in: $T = T_{in}$
3 out: q · n = convective flux
4 RHS: for constant temperature boundary at the tube wall: = Td

for flux at the wall : = ks * (Td-T)/(a ln(d/a)) = 776.098 * (Td-T)

where:

ks = 1.12472 J/(s-m-K) from (3)
a = 0.01095 m
d = 0.0125 m

Mesh

Initial
*Refine *2*
Refine Inlet and Outlet

Alternate:

Parameters

Max edge size, general (3e-3)

Solve

Solver Parameters
Nonlinear
Maximum Number of iterations 200

Post
Plot Parameters
ArrowLine

Geometry Boundaries

C*ross section plot parameters*

General

Line

Line

The two point option allows the plotting of center line and radial temperatures and velocities.

Executing FEMLAB

Execution of FEMLAB is started by clicking on *Solve*. There were three issues, however, that were needed to be resolved before successful convergence could take place.

1. *Memory*

   Initially 128MB memory was used on a 600 megahertz Intel Pentium III PC. For this problem 128 MB (or even 256 MB) seemed inadequate. A message "Out of memory" was often encountered. Upgrading memory to 512 MB allowed solutions to be obtained, though additional memory is clearly desirable for more detailed investigations.

2. *Mesh Size*

   Choosing an adequate mesh size is critical. Picking a large mesh size often leads to the message "Step Size Too Small" and the run is terminated. Choosing a small mesh size results in very long run times (hours). Selecting the initial mesh and refining twice is often adequate though trial and error may be required.

3. *Approach to the Solution*

   Attempting to solve both the momentum and energy equations at one time is often unsuccessful even if very long execution times are allowed. The most successful procedure is to:

   a. Solve the Navier-Stokes equation alone. This is done by:

      Multiphysics

         Solve for variables

            Highlighting Navier Stokes

   b. Once the Navier Stokes equation is solved then both the Navier-Stokes and the Energy Equation are solved together. This is done by

Multiphysics

Solve for variables

Highlighting both the Navier-Stokes and Energy Equation. To do this hold the *Ctrl* key down.

c. Modifying the volume force by the factor “fact” specified in:

Options

Add/Edit Constants

set fact to new value

The parameter fact is initially set to 100 and both equations are solved. Then the value of fact is reduced to 50, 20 and 10. Finally fact is set to 1 which is the solution to the problem..

FEMLAB Results

The value of z at which the center line velocity fell to zero is entered into the table of Figure 4B which then calculates the values of Gr, Re and Z* x 10^3 for plotting.

In all ten runs are selected from Reference (3). For each run FEMLAB simulations are made using a constant temperature wall boundary condition and for using a flux boundary condition which recognizes the resistance of the wall.

Figures 5 to 9 are plots taken from FEMLAB that illustrates the velocity and temperature profiles encountered for run 24. Similar profiles are encountered for both the constant temperature and flux runs.

Figure 10A summarizes all the FEMLAB simulations and compares the results to the experimental values given in (3). The data are plotted in Figure 10B in dimensionless form.

Discussion of Results

The plot of Figure 10B illustrates two trends:

1. Constant temperature boundary simulations fall below the null point experimental values.

   Experimentally, the greater the heating with a fixed flow, the closer to the entrance flow reversal takes place. In the experiments the wall temperature at the entrance and downstream must be less than Td due to the thermal resistance of the wall. Setting the wall temperature to Td in the simulations results in greater heating to the fluid causing the fluid to have flow reversal nearer the entrance. The FEMLAB simulations are therefore consistent with experimentally observed values.

Figure 11 is a plot of the wall temperature near the entrance for run 24. The wall temperature becomes steady (equal to Td) only after rising from an average of Td and To and going through an excursion. This is a result of the discontinuity.

2. Flux boundary condition simulations fall above the null point experimental values.

   Experimentally, the temperature of the fluid at the wall (at the entrance) is probably warmer than To (due to heat leakage). As a result the temperature of the fluid along the wall as it moves downstream will be warmer than the simulation which starts the fluid temperature at the wall at To.

   Figure 12 is a plot of the wall temperature for run 24 with a flux boundary. The boundary condition in the simulation forces the fluid temperature at the wall to be To even though the wall temperature is Td (less the resistance drop). Also, since the inlet fluid temperature is uniform the fluid flux must be initially zero at the wall. There is a discontinuity both in the temperature and the flux.

   Since the temperature of fluid along the wall at the entrance (and downstream) is less than the experimental conditions, the point of inversion in the simulation falls downstream of the experimental values (Figure 10B).

   It is unclear whether a finer mesh size would bring the simulated results closer to the experimental values. Memory limitations prevented testing this.

Conclusions

FEMLAB has a wide range of capabilities, is very well supported and has extensive documentation, though this investigator found the documentation difficult to follow at times. FEMLAB gives quite reasonable results when compared to experiment. To get better simulations, however, finer mesh points may be needed.

Nomenclature

English

a	Inside tube radius
b	Temperature coefficient of viscosity
bo	Temperature coefficient of viscosity at To

alpha	Coefficient of expansion
Cp	Femlab Heat capacity
Cpo	Heat Capacity at To (used in simulation)
d	Outside radius of tube
F	Femlab vector of body forces
Fz	Body force in z direction
FY	Body force in z direction (usedin simulation)
g	acceleration of gravity
Gr	Grashof number = $a^3 r_o^2 g\ b_o$ (Td – To)/ m_o^2
Gz	Graetz number = p Re Pr/Z
k	Femlab Thermal conductivity
kc	Thermal conductivity of fluid at To (used in simulation)
ko	Equal to kc
ks	Thermal conductivity of glass tubing (used in simulation)
mu	Viscosity (used in simulation)
muo	Viscosity at To (used in simulation)
p	FEMLAB pressure
Pr	Prandtl Number = Cpo m_o /kc
Psi	Parameter = bo (Td-To)
Q	FEMLAB heat flux
r	FEMLAB radial distance
rho0	Density of water at To (used in simulation)
Re	Reynolds number = $a\ v_{avg}\, r_o / m_o$

s	FEMLAB parameter $0 \le s \le 1$
T, t	FEMLAB Temperature
Td, t_d	Outside wall temperature (used in simulation)
To, t_o, Tin	Entering fluid temperature (used in simulation)
u	FEMLAB velocity vector
u	FEMLAB velocity in r direction
Uin	Equal to v_{max}
v	FEMLAB velocity in z direction
v_{avg}	Average z velocity in entering fluid (used in simulation)
v_{max}	Maximum z velocity in entering fluid (used in simulation)
z	Distance downstream
Z	z/a
Z*	Z/(p Re Pr)
Other	
b	Same as alpha
h	FEMLAB symbol for viscosity
m	same as mu (viscosity)
m_o	Viscosity at temperature To
r_o	Density at temperature To
∇	Vector differential operator (divergence)

References

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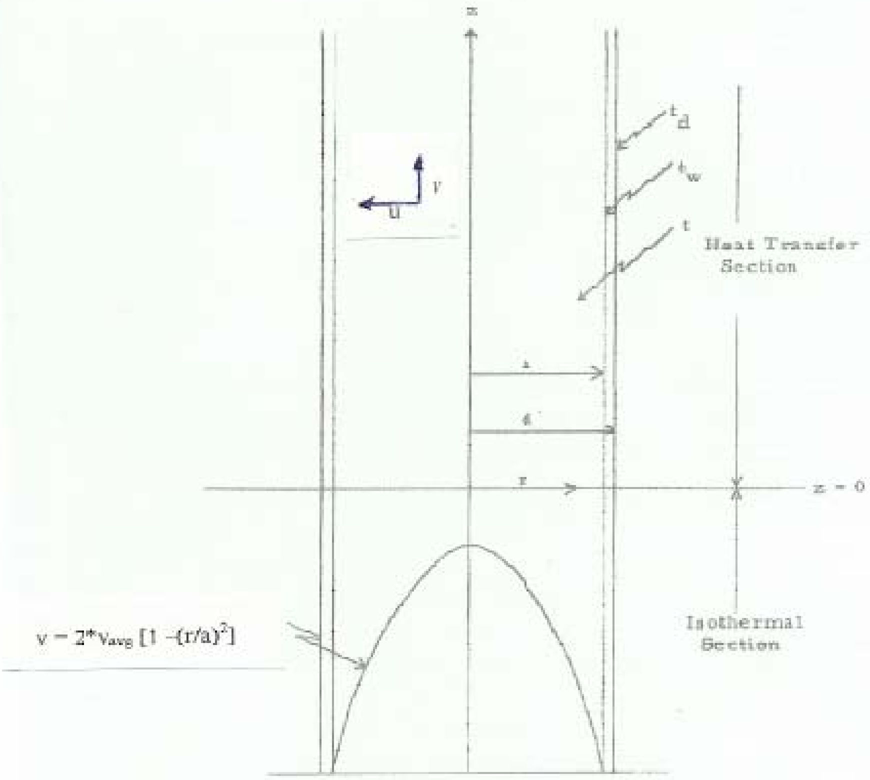
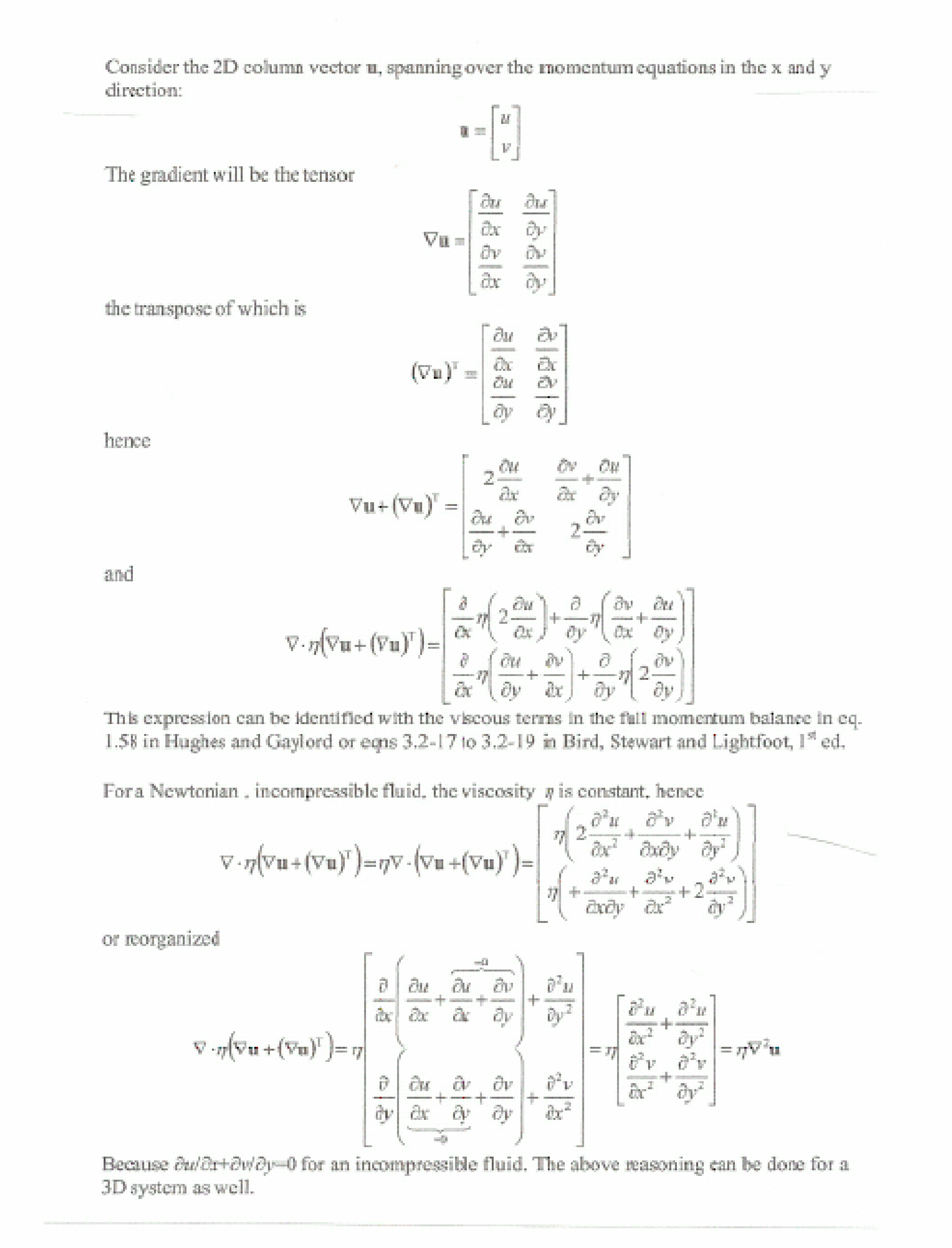


Figure 1. Definition of Coordinate System and Nomenclature



Consider the 2D column vector $\mathbf{u}$, spanning over the momentum equations in the x and y direction:

$$\mathbf{u}=\begin{bmatrix} u \\ v \end{bmatrix}$$

The gradient will be the tensor

$$\nabla\mathbf{u}=\begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{bmatrix}$$

the transpose of which is

$$(\nabla\mathbf{u})^T=\begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial v}{\partial x} \\ \dfrac{\partial u}{\partial y} & \dfrac{\partial v}{\partial y} \end{bmatrix}$$

hence

$$\nabla\mathbf{u}+(\nabla\mathbf{u})^T=\begin{bmatrix} 2\dfrac{\partial u}{\partial x} & \dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y} \\ \dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x} & 2\dfrac{\partial v}{\partial y} \end{bmatrix}$$

and

$$\nabla\cdot\eta\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)=\begin{bmatrix} \dfrac{\partial}{\partial x}\eta\left(2\dfrac{\partial u}{\partial x}\right)+\dfrac{\partial}{\partial y}\eta\left(\dfrac{\partial v}{\partial x}+\dfrac{\partial u}{\partial y}\right) \\ \dfrac{\partial}{\partial x}\eta\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right)+\dfrac{\partial}{\partial y}\eta\left(2\dfrac{\partial v}{\partial y}\right) \end{bmatrix}$$

This expression can be identified with the viscous terms in the full momentum balance in eq. 1.58 in Hughes and Gaylord or eqns 3.2-17 to 3.2-19 in Bird, Stewart and Lightfoot, 1st ed.

For a Newtonian , incompressible fluid, the viscosity η is constant, hence

$$\nabla\cdot\eta\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)=\eta\nabla\cdot\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)=\begin{bmatrix} \eta\left(2\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 v}{\partial x\partial y}+\dfrac{\partial^2 u}{\partial y^2}\right) \\ \eta\left(+\dfrac{\partial^2 u}{\partial x\partial y}+\dfrac{\partial^2 v}{\partial x^2}+2\dfrac{\partial^2 v}{\partial y^2}\right) \end{bmatrix}$$

or reorganized

$$\nabla\cdot\eta\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)=\eta\begin{bmatrix} \dfrac{\partial}{\partial x}\left(\dfrac{\partial u}{\partial x}+\overbrace{\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}}^{=0}\right)+\dfrac{\partial^2 u}{\partial y^2} \\ \dfrac{\partial}{\partial y}\left(\underbrace{\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}}_{=0}+\dfrac{\partial v}{\partial y}\right)+\dfrac{\partial^2 v}{\partial x^2} \end{bmatrix}=\eta\begin{bmatrix} \dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2} \\ \dfrac{\partial^2 v}{\partial x^2}+\dfrac{\partial^2 v}{\partial y^2} \end{bmatrix}=\eta\nabla^2\mathbf{u}$$

Because $\partial u/\partial x+\partial v/\partial y=0$ for an incompressible fluid. The above reasoning can be done for a 3D system as well.

Figure 2 Relationship to 2D Cartesian Coordinate System

v_r, v_θ, and v_z are the velocities in the r, θ, and z directions respectively. In the following section:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}$$

$$\rho\left[\frac{Dv_r}{Dt} - \frac{v_\theta^2}{r}\right]$$

$$= -\frac{\partial P}{\partial r} + F_r + 2\frac{\partial}{\partial r}\left(\mu\frac{\partial v_r}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)\right]$$

$$+ \frac{\partial}{\partial z}\left[\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)\right] + \frac{2\mu}{r}\left(\frac{\partial v_r}{\partial r} - \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} - \frac{v_r}{r}\right)$$

$$\rho\left[\frac{Dv_\theta}{Dt} + \frac{v_r v_\theta}{r}\right]$$

$$= -\frac{1}{r}\frac{\partial P}{\partial \theta} + F_\theta + \frac{2}{r}\frac{\partial}{\partial \theta}\left(\frac{\mu}{r}\frac{\partial v_\theta}{\partial \theta}\right) + \frac{\partial}{\partial z}\left[\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)\right]$$

$$+ \frac{\partial}{\partial r}\left[\mu\left(\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right)\right] + \frac{2\mu}{r}\left[\frac{1}{r}\frac{\partial v_r}{\partial \theta} + \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r}\right]$$

$$\rho\frac{Dv_z}{Dt}$$

$$= -\frac{\partial P}{\partial z} + F_z + 2\frac{\partial}{\partial z}\left(\mu\frac{\partial v_z}{\partial z}\right) + \frac{1}{r}\frac{\partial}{\partial r}\left[\mu r\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)\right]$$

$$+ \frac{1}{r}\frac{\partial}{\partial \theta}\left[\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)\right]$$

Figure 3. Navier-Stokes Equation of Motion
for An Incompressible Fluid-
Cylindrical Coordinates

Figure 4A

Physical Properties of Water for Experiments - Flux and Constant Temperature Results

	Run No ==>	Run 1	Run 6	Run 24	Run 19	Run 35	Run 2	Run_28	Run 30	Run 23	Run 3
Inlet, Wall and Reference											
Temperatures	Units										
td F (Td)		102	109.2	107.5	108.4	114.8	102	117	116.8	108.3	131.5
td K (Td K)	Kelvins	312.0388889	316.0388889	315.0944444	315.5944444	319.15	312.03889	320.372222	320.26111	315.5388889	328.4277778
to F (To)		80.2	80.9	82.1	81.2	82.2	80.3	71.4	73.4	82.3	78.5
to K (To K)	Kelvins	299.9277778	300.3166667	300.9833333	300.4833333	301.03889	299.98333	295.038889	296.15	301.0944444	298.9833333
to C (To C)		26.77777778	27.16666667	27.83333333	27.33333333	27.888889	26.833333	21.8888889	23	27.94444444	25.83333333
Density											
low T	C	26.7	27.1	27.8	27.3	27.8	26.8	21.8	23	27.9	25.8
low rho	gms/milliter	0.9966243	0.9965146	0.996319	0.9964591	0.996319	0.996597	0.9988444	0.9975674	0.9962907	0.9968657
high T	C	26.8	27.2	27.9	27.4	27.9	26.9	21.9	23.1	28	25.9
high rho	gms/milliter	0.996597	0.9964869	0.9962907	0.9964313	0.9962907	0.9965696	0.9988219	0.9975437	0.9962623	0.9968393
rho	grams/milliter	0.996603067	0.996496133	0.996309567	0.996449833	0.9962938	0.9965879	0.9988244	0.9975674	0.996278078	0.9968569
rho0	**kg/m^3**	**996.6030667**	**996.4961333**	**996.3095667**	**996.4498333**	**996.29384**	**996.58787**	**998.8244**	**997.5674**	**996.2780778**	**996.8569**
Dynamic Viscosity											
low T	C	26	27	27	27	27	26	21	23	27	25
low mu	centipoise	0.8737	0.8545	0.8545	0.8545	0.8545	0.8737	0.981	0.9358	0.8545	0.8937
high T	C	27	28	28	28	28	27	22	24	28	26
hgh Mu	centipoise	0.8545	0.836	0.836	0.836	0.836	0.8545	0.9579	0.9142	0.836	0.8737
mu	centipoise	0.858766667	0.851416667	0.839083333	0.848333333	0.8380556	0.8577	0.96046667	0.9358	0.837027778	0.877033333
muo	**kg/m-s**	**0.000858767**	**0.000851417**	**0.000839083**	**0.000848333**	**0.0008381**	**0.0008577**	**0.00096047**	**0.0009358**	**0.000837028**	**0.000877033**
Thermal Conductivity											
low T	C	0	0	0	0	0	0	0	0	0	0
low k	BTU/hr-ft-F	0.343	0.343	0.343	0.343	0.343	0.343	0.343	0.343	0.343	0.343
high T	C	37.77777778	37.77777778	37.77777778	37.77777778	37.777778	37.777778	37.7777778	37.777778	37.77777778	37.77777778
high k	BTU/hr-ft-F	0.363	0.363	0.363	0.363	0.363	0.363	0.363	0.363	0.363	0.363
k	BTU/hr-ft-F	0.357176471	0.357382353	0.357735294	0.357470588	0.3577647	0.3572059	0.35458824	0.3551765	0.357794118	0.356676471
ko	**J/s-m-K**	**0.618022447**	**0.618378685**	**0.618989379**	**0.618531359**	**0.6190403**	**0.6180733**	**0.61354402**	**0.6145618**	**0.619091162**	**0.617157297**
Heat Capacity											
low T	C	26	27	27	27	27	26	21	23	27	25
low Cp	cal/gm-C	0.99885	0.99878	0.99878	0.99878	0.99878	0.99885	0.99933	0.99912	0.99878	0.99892
high T	C	27	28	28	28	28	27	22	24	28	26
high Cp	cal/gm-C	0.99878	0.99873	0.99873	0.99873	0.99873	0.99878	0.99921	0.99902	0.99873	0.99885
Cp	cal/gm-C	0.998795556	0.998771667	0.998738333	0.998763333	0.9987356	0.9987917	0.99922333	0.99912	0.998732778	0.998861667
Cpo	**J/kg-K**	**4178.960604**	**4178.860653**	**4178.721187**	**4178.825787**	**4178.7096**	**4178.9443**	**4180.75043**	**4180.3181**	**4178.697942**	**4179.237213**

Figure 4B												
	Run No ==>	Run 1	Run 6	Run 24	Run 19	Run 35	Run 2	Run_28	Run 30	Run 23	Run 3	
Coefficient of Expansion												
low T	C	26	27	27	27	27	26	21	23	27	25	
low beta	1/F	0.14822	0.15343	0.15343	0.15343	0.15343	0.14822	0.12054	0.13194	0.15343	0.14289	
high T	C	27	28	28	28	28	27	22	24	28	26	
high beta	1/F	0.15343	0.15859	0.15859	0.15859	0.15859	0.15343	0.12628	0.13746	0.15859	0.14822	
beta	1/F	0.152272222	0.15429	0.15773	0.15515	0.1580167	0.1525617	0.12564222	0.13194	0.158303333	0.147331667	
betao (alpha)	**1/Kelvins**	**0.00027409**	**0.000277722**	**0.000283914**	**0.00027927**	**0.0002844**	**0.0002746**	**0.00022616**	**0.0002375**	**0.000284946**	**0.000265197**	
Velocity												
a	**m**	**0.01095**	**0.01095**	**0.01095**	**0.01095**	**0.01095**	**0.01095**	**0.01095**	**0.01095**	**0.01095**	**0.01095**	
gms/sec	gm/s	5.18	1.97	4.74	7.31	4.21	9.82	12.2	10.2	3.88	10.95	
v(average)	m/s	0.013798434	0.00524823	0.012630086	0.019475302	0.011218	0.0261588	0.03242597	0.0271444	0.010338878	0.029161077	
Uin -v(maximum)	m/s	0.027596868	0.01049646	0.025260171	0.038950603	0.0224361	0.0523176	0.06485194	0.0542888	0.020677756	0.058322153	
Temp Coeff of Viscosity												
low Temperature	C	26	27	27	27	27	26	21	23	27	25	
low b	1/F	0.012418	0.012285	0.012285	0.012285	0.012285	0.012418	0.013302	0.012946	0.012285	0.012616	
high Temperature	C	27	28	28	28	28	27	22	24	28	26	
high b	1/F	0.012285	0.012116	0.012116	0.012116	0.012116	0.012285	0.013139	0.012781	0.012116	0.012418	
b	**1/Kelvins**	0.0221662	0.0220623	0.0218595	0.0220116	0.0218426	0.0221529	0.0236828	0.0233028	0.0218257	0.0224118	
Parameters												
Re	Reynolds Number	175.3440187	67.26055612	164.213789	250.48817	146.0312	332.82233	369.244392	316.84985	134.7498312	362.9396318	
Gr	Grashof Number	57581.45373	77037.56027	72750.80382	74991.12394	93769.4	57567.437	79804.1559	83810.029	75102.80873	129932.195	
Gr/Re		328.3913199	1145.360144	443.0249389	299.3799026	642.11893	172.96747	216.128281	264.51024	557.3499282	357.9994677	
ln (Gr/Re)		5.794205946	7.043474403	6.093626064	5.701713344	6.4647735	5.1531036	5.37587213	5.5778799	6.32319328	5.880531499	
Pr		5.806831266	5.753677629	5.664548406	5.731378302	5.6571292	5.7991185	6.54471606	6.3654157	5.649711171	5.939053725	
Psi		0.268457311	0.346868383	0.308461833	0.332619733	0.3955938	0.2670655	0.59996427	0.5618564	0.315260111	0.659903	
Femlab - Boundary Flux												
z	m	0.285	0.061	0.22	0.43	0.16	0.931	0.87	0.624	0.161	0.573	
Z		26.02739726	5.570776256	20.0913242	39.26940639	14.611872	85.022831	79.4520548	56.986301	14.70319635	52.32876712	
Z*		0.008136752	0.004582054	0.006875181	0.008706799	0.0056301	0.0140221	0.01046528	0.0089937	0.006147631	0.007727506	
Gz		122.8991637	218.2427217	145.4507143	114.8527675	177.61735	71.316231	95.5540482	111.18836	162.6642911	129.4078588	
Z**1000		**8.136751873**	**4.582054293**	**6.875181086**	**8.706799334**	**5.6300805**	**14.022053**	**10.4652814**	**8.9937474**	**6.147630764**	**7.72750596**	
Femlab - Constant Temp												
z	m	0.2185	0.052	0.171	0.324	0.1265	0.668	0.62	0.455	0.122	0.41	
Z		19.9543379	4.748858447	15.61643836	29.5890411	11.552511	61.004566	56.6210046	41.552511	11.14155251	37.44292237	
Z*		0.006238176	0.003906013	0.005343891	0.006560472	0.0044513	0.0100609	0.00745802	0.0065579	0.004658453	0.00552928	
Gz		160.3032569	256.0155005	187.129574	152.4280557	224.65436	99.394328	134.083906	152.48689	214.6635317	180.8553734	
Z**1000		**6.238176436**	**3.906013496**	**5.343890753**	**6.560472057**	**4.4512824**	**10.060936**	**7.45801661**	**6.5579408**	**4.658453125**	**5.529280007**	

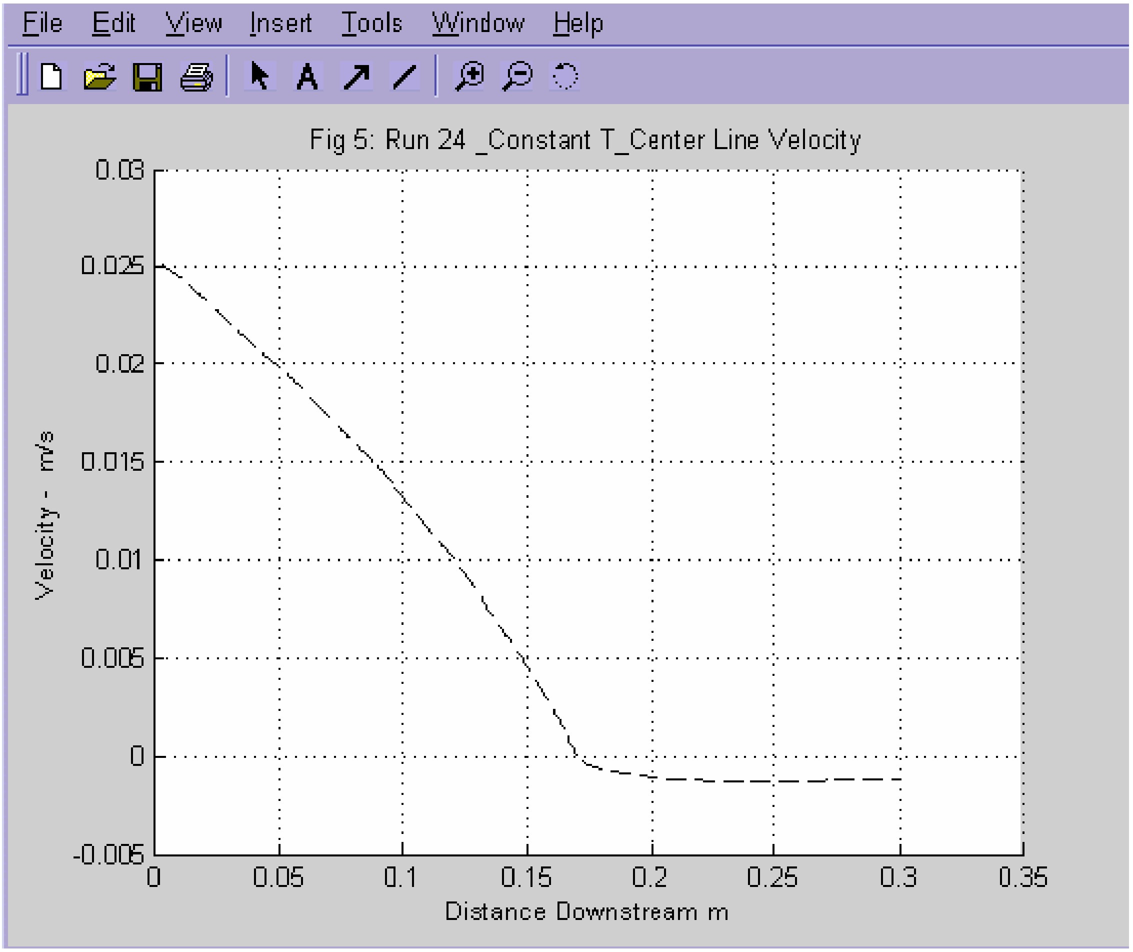

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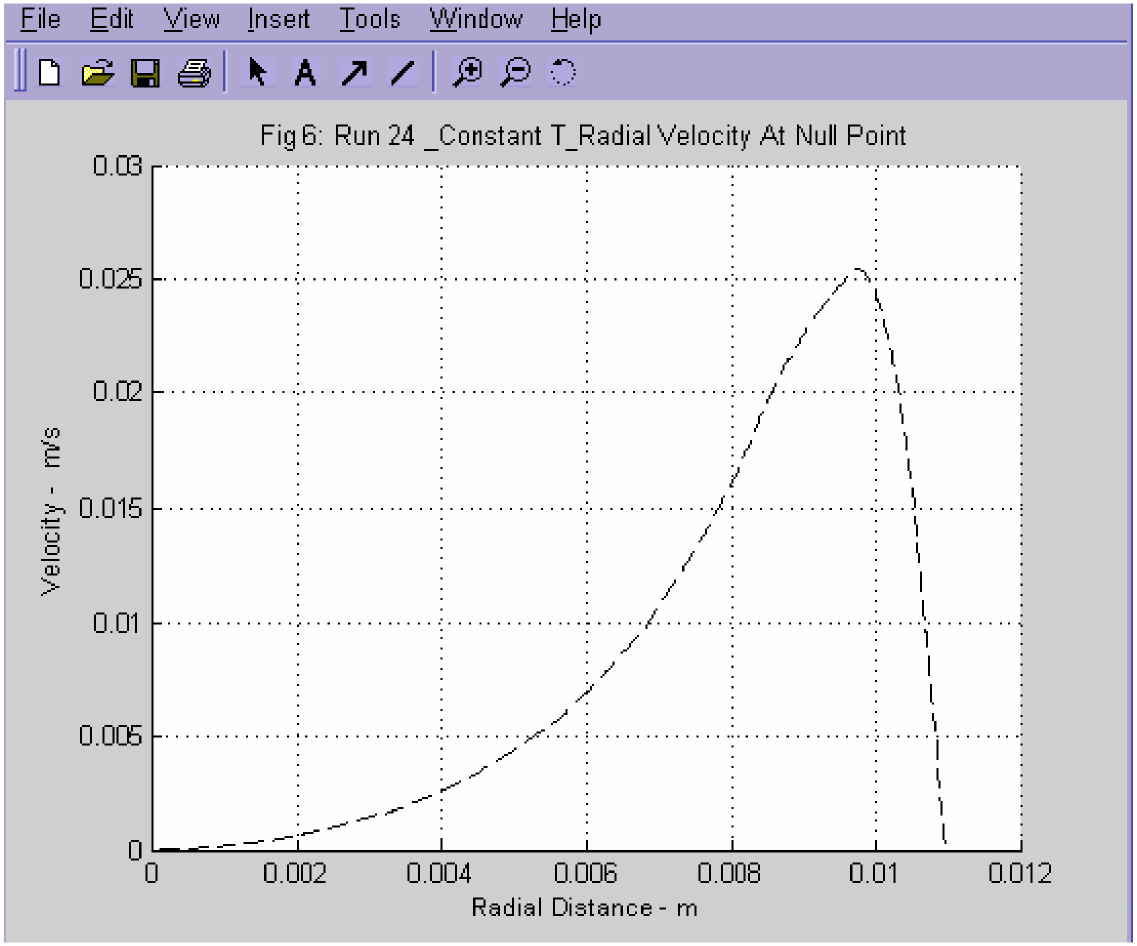
Fig 5: Run 24 _Constant T_Center Line Velocity
Velocity - m/s
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Distance Downstream m


File
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Tools
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Help
Fig 6: Run 24 _Constant T_Radial Velocity At Null Point
Velocity - m/s
0.03
0.025
0.02
0.015
0.01
0.005
0
0
0.002
0.004
0.006
0.008
0.01
0.012
Radial Distance - m

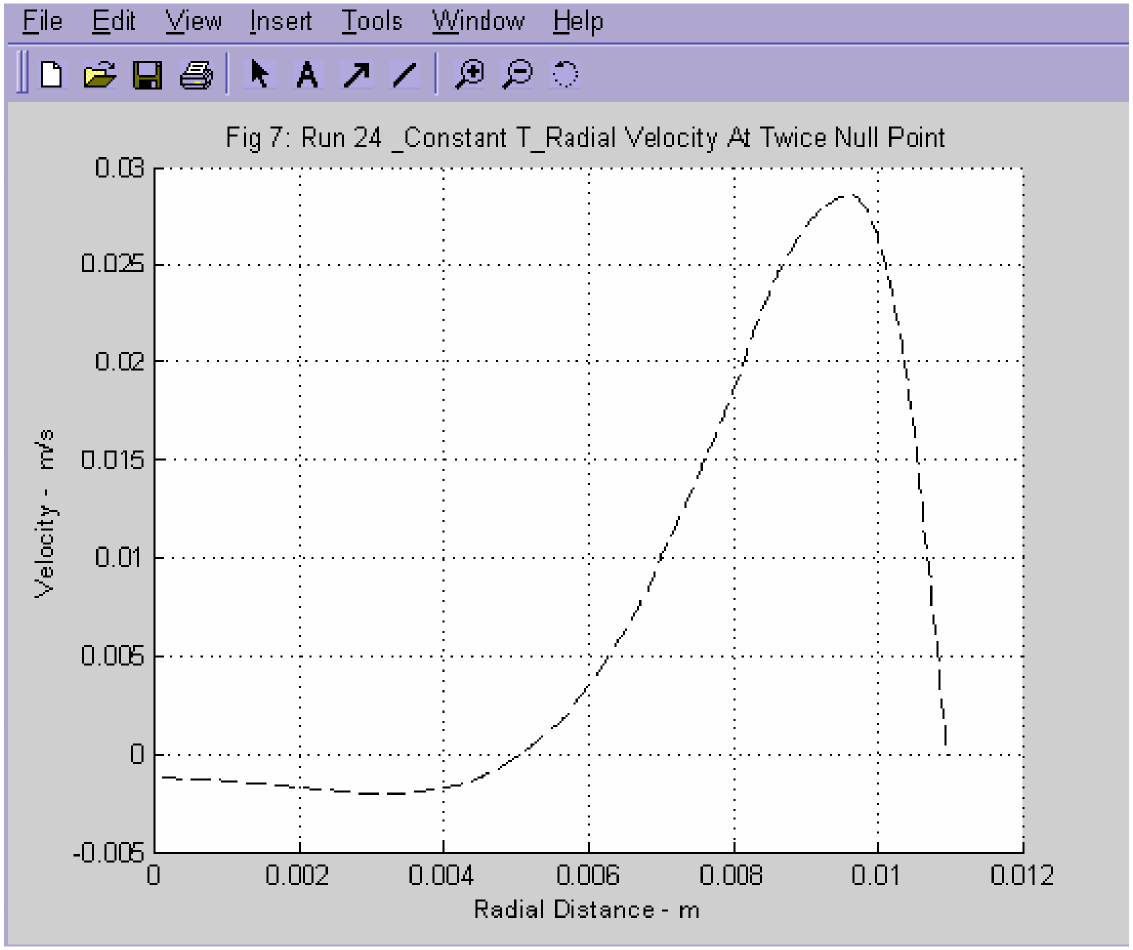
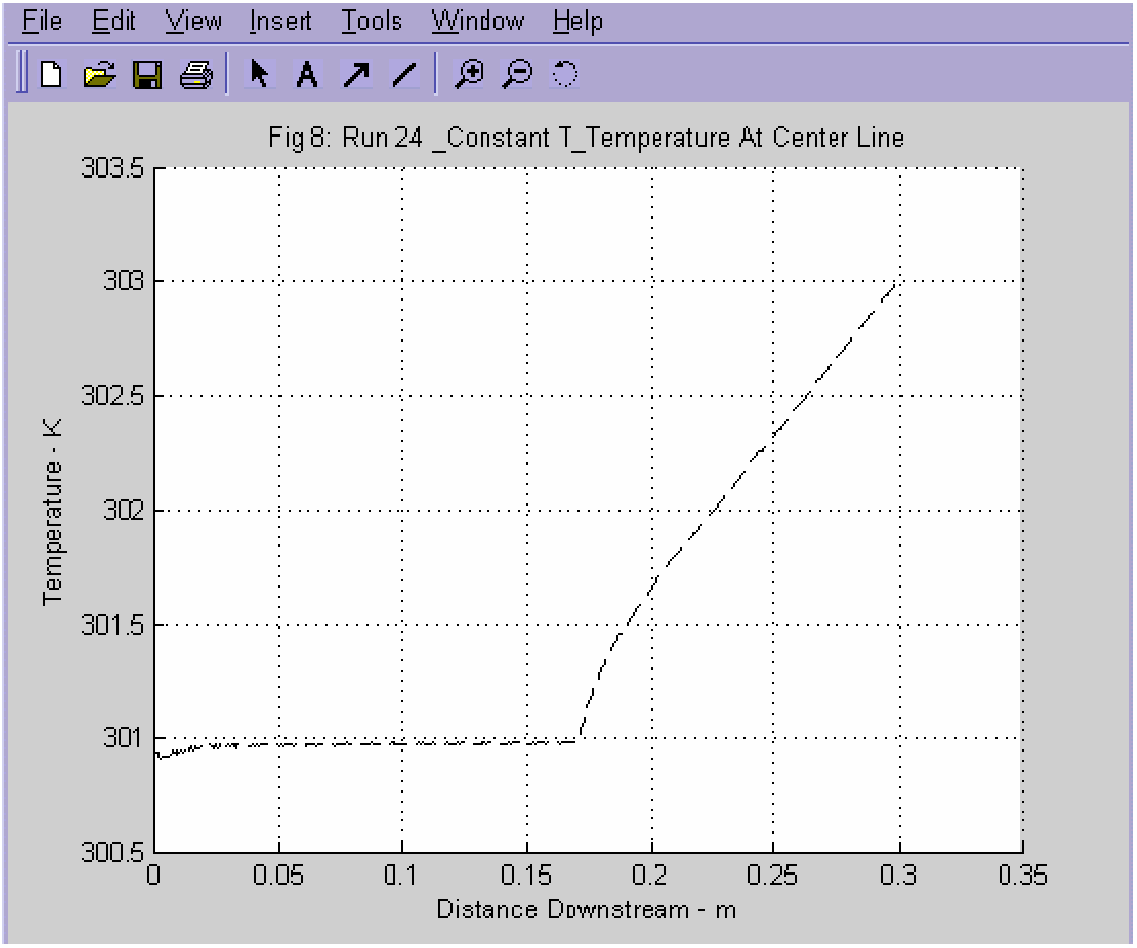


Fig 7: Run 24 _Constant T_Radial Velocity At Twice Null Point


File
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View
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Tools
Window
Help
Fig 8: Run 24 _Constant T_Temperature At Center Line
303.5
303
302.5
302
301.5
301
300.5
Temperature - K
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
Distance Downstream - m

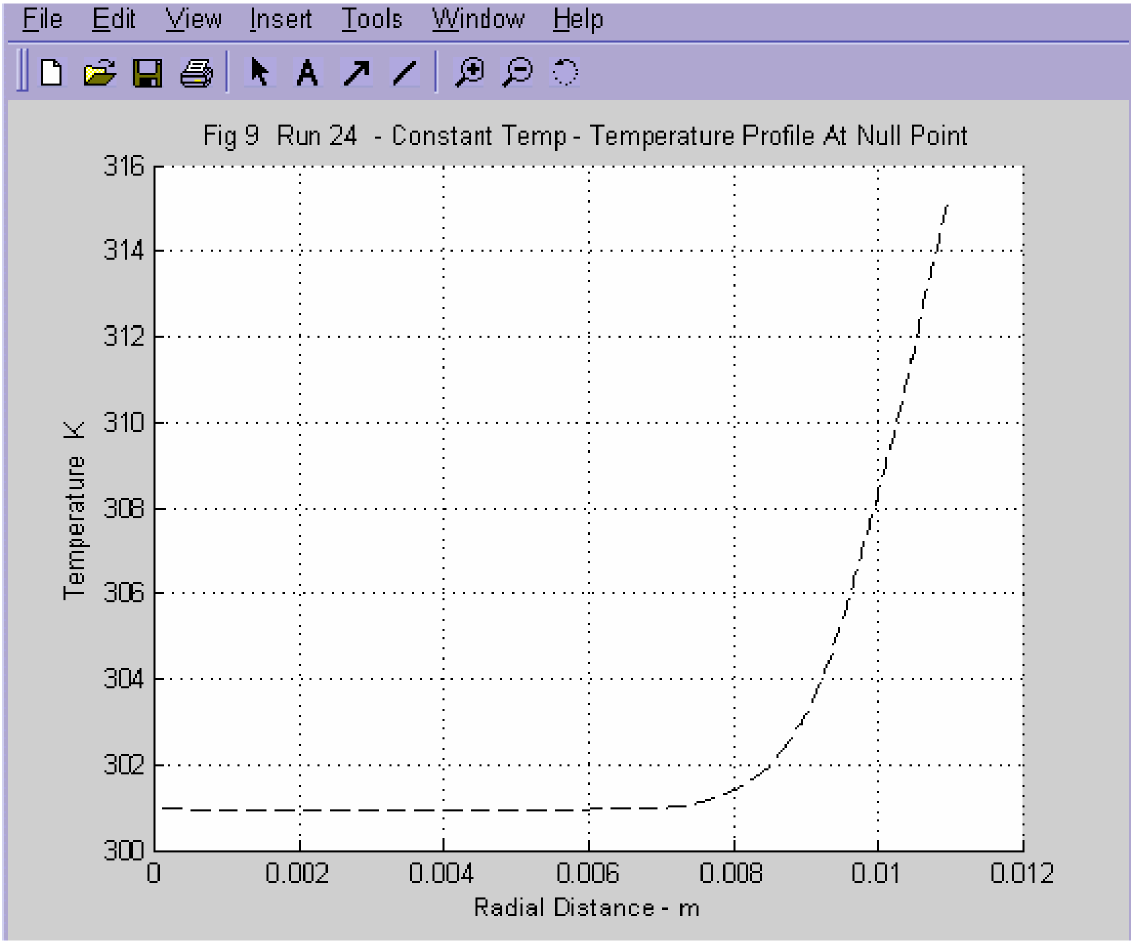


Fig 9 Run 24 - Constant Temp - Temperature Profile At Null Point

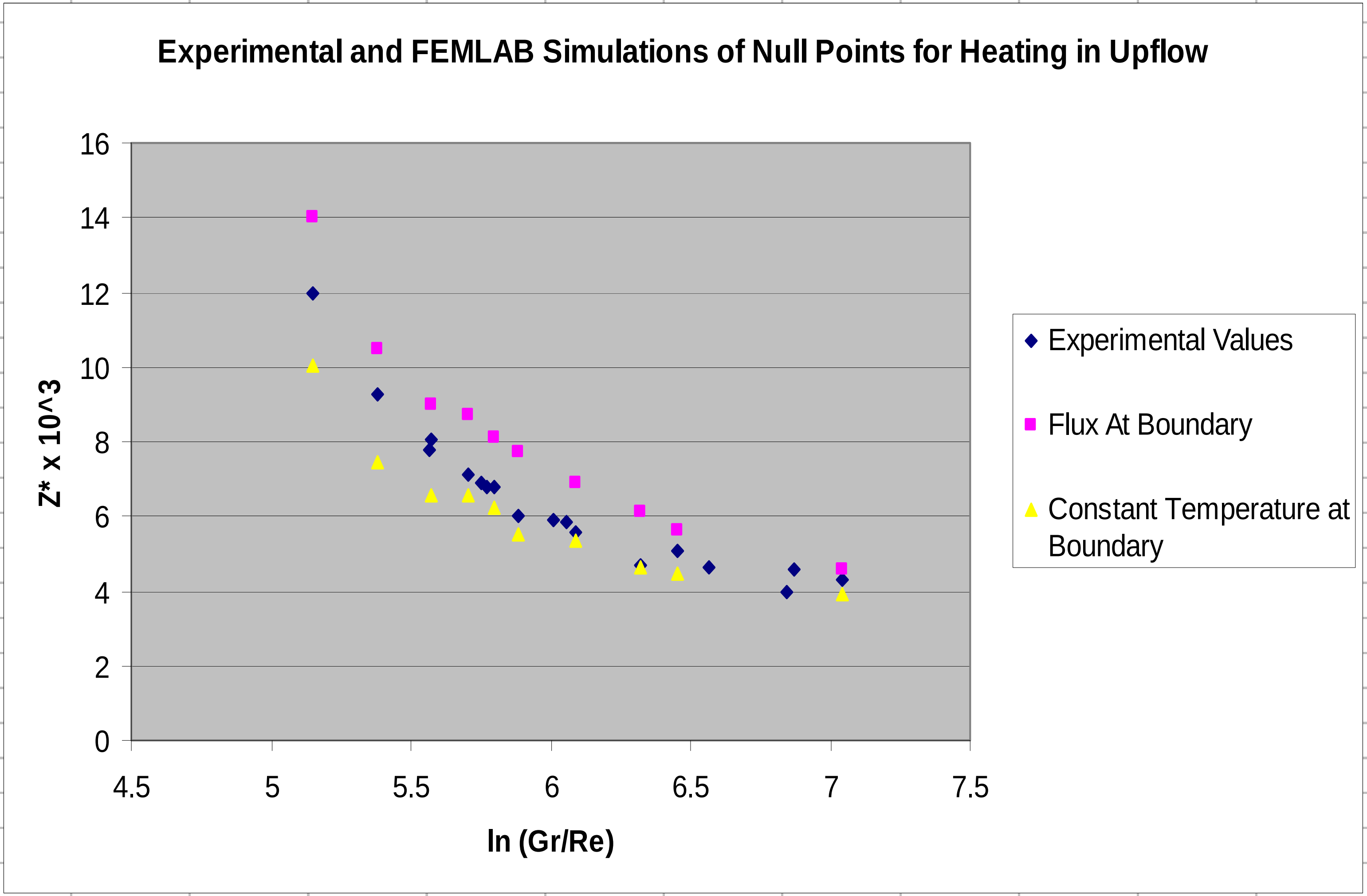
Figure 10A   Results of Study - Experimental Data From Page 243 Thesis

						Experimental---------->		Femlab-Flux------------->		Femlab-Temp
Run	Pr	Gr	Re	Gznull	Gr/Re	ln(Gr/Re)	Z* x 10^3	ln(Ge/Re)	Z* x 10^3	Z* x 10^3
1	5.79	57500	175	147	328.5714286	5.794754253	6.8027211	5.79675	8.136	6.238
2	5.78	57500	333	83.7	172.6726727	5.151397737	11.947431	5.151	14.02	10.06
3	5.93	130000	362	166	359.1160221	5.883645518	6.0240964	5.8805	7.72	5.528
6	5.74	76900	67.3	233	1142.644874	7.041100919	4.2918455	7.043	4.582	3.906
15	5.85	66800	156	171	428.2051282	6.059602352	5.8479532			
16	5.81	66500	254	129	261.8110236	5.56762296	7.751938			
19	5.72	75000	250	140	300	5.703782475	7.1428571	5.7	8.706	6.56
20	5.71	74000	231	147	320.3463203	5.769402662	6.8027211			
21	5.69	75600	186	170	406.4516129	6.007464888	5.8823529			
23	5.66	74400	134	214	555.2238806	6.319371421	4.6728972	6.23	6.147	4.658
24	5.67	72500	164	180	442.0731707	6.091475413	5.5555556	6.093	6.875	5.34
25	5.9	76100	242	145	314.4628099	5.750865818	6.8965517			
26	5.88	77000	108	216	712.962963	6.569429474	4.6296296			
27	5.87	76400	81.2	251	940.8866995	6.846822728	3.9840637			
28	6.53	79900	368	108	217.1195652	5.380448194	9.2592593	5.375	10.46	7.458
30	6.38	83600	317	124	263.7223975	5.574897025	8.0645161	5.5748	8.99	6.55
34	5.69	95900	99.8	219	960.9218437	6.867893078	4.56621			
35	5.66	93000	146	198	636.9863014	6.45674815	5.0505051	6.465	5.63	4.451

<------------------------- Plot Data--->

ln(Gr/Re)	Experimental	Z* x 10^3 Flux	Z* x 10^3 Temp
5.79475425	6.802721088	8.136	6.238
5.15139774	11.9474313	14.02	10.06
5.88364552	6.024096386	7.72	5.528
7.04110092	4.291845494	4.58	3.906
6.05960235	5.847953216		
5.56762296	7.751937984		
5.70378247	7.142857143	8.706	6.56
5.76940266	6.802721088		
6.00746489	5.882352941		
6.31937142	4.672897196	6.147	4.658
6.09147541	5.555555556	6.875	5.34
5.75086582	6.896551724		
6.56942947	4.62962963		
6.84682273	3.984063745		
5.38044819	9.259259259	10.46	7.458
5.57489703	8.064516129	8.99	6.55
6.86789308	4.566210046		
6.45674815	5.050505051	5.63	4.451

Figure 10 B Plot of Results



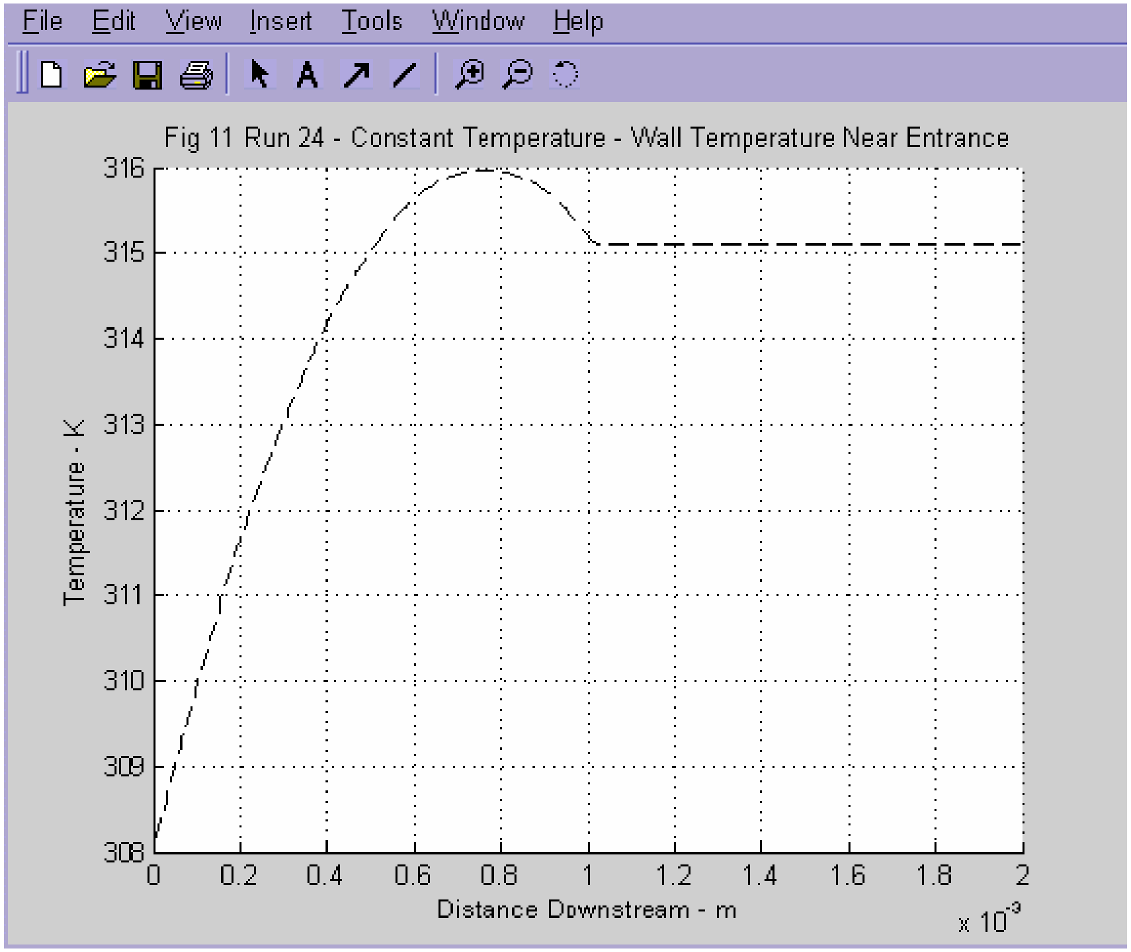

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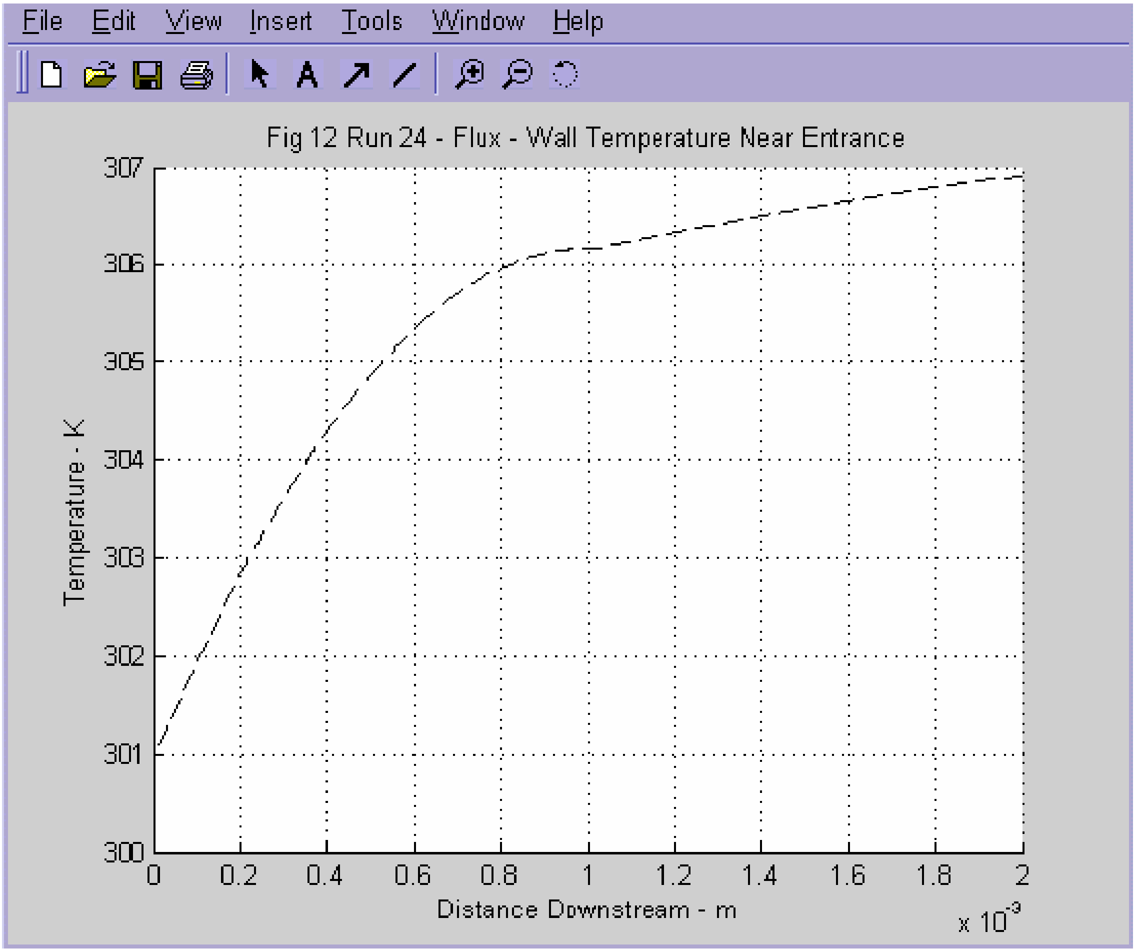
Fig 11 Run 24 - Constant Temperature - Wall Temperature Near Entrance
Temperature - K
316
315
314
313
312
311
310
309
308
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Distance Downstream - m
x 10-3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View
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Tools
Window
Help
Fig 12 Run 24 - Flux - Wall Temperature Near Entrance
Temperature - K
307
306
305
304
303
302
301
300
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Distance Downstream - m
x 10^{-3}