Why Did the World Trade Center Towers Collapse?

A Numerical Heat Transfer Analysis

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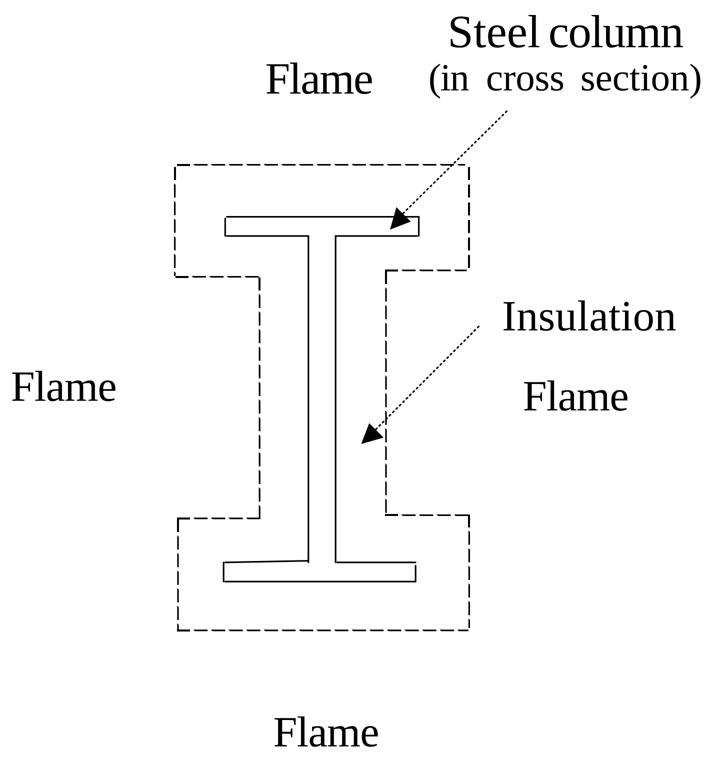
Radiative Heating of Insulated Steel



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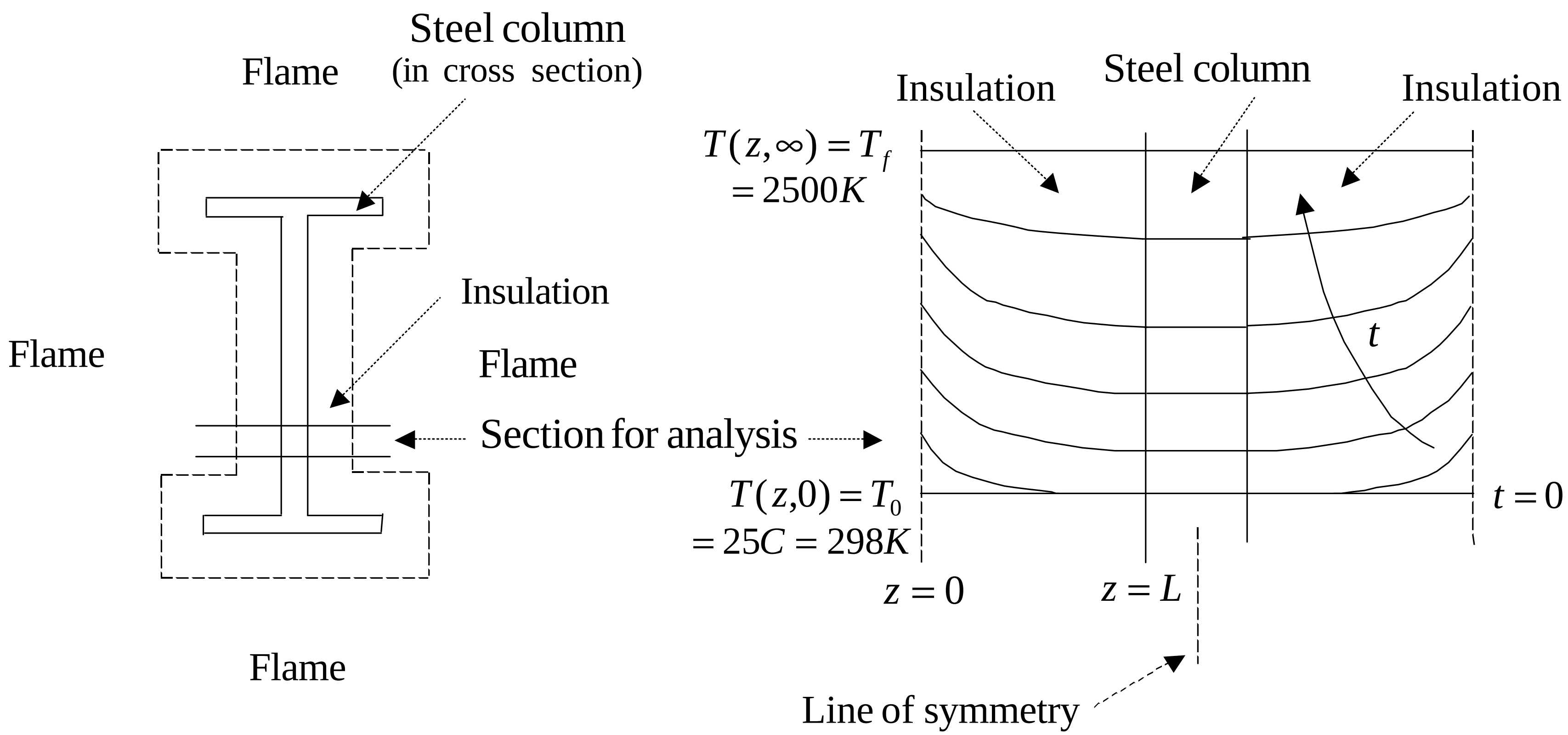
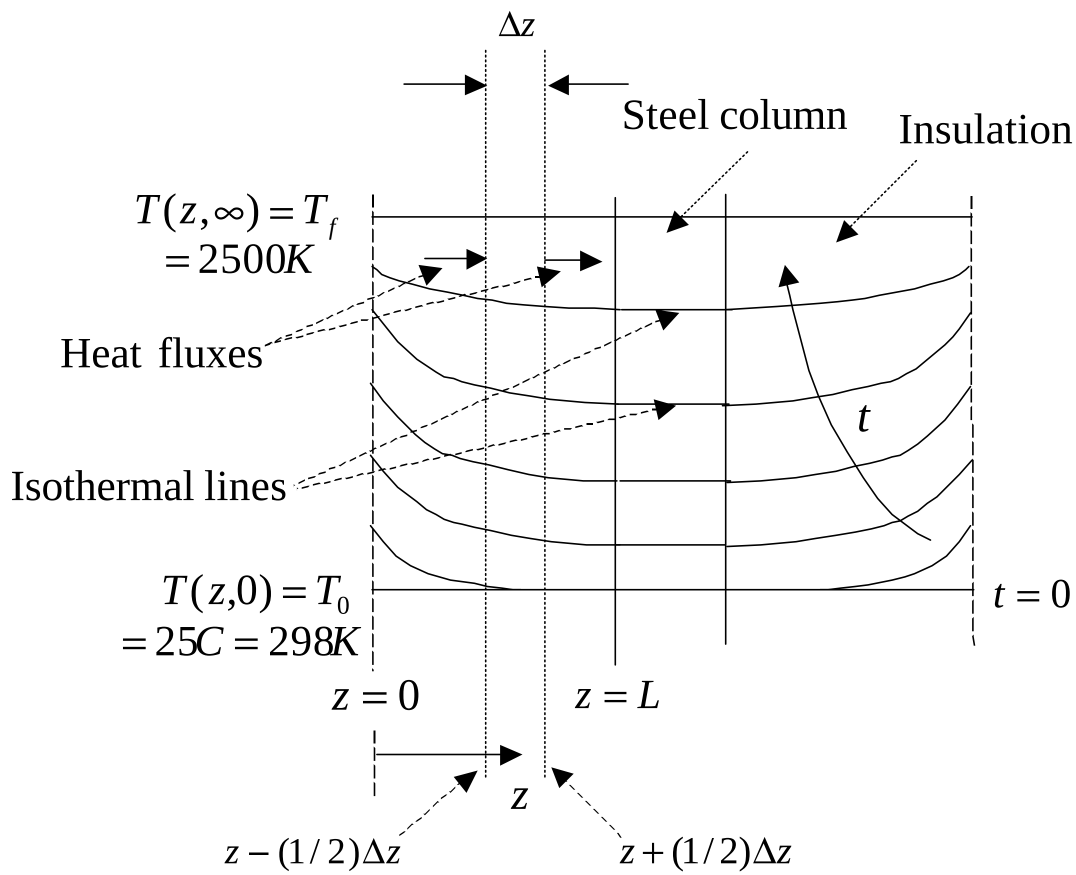
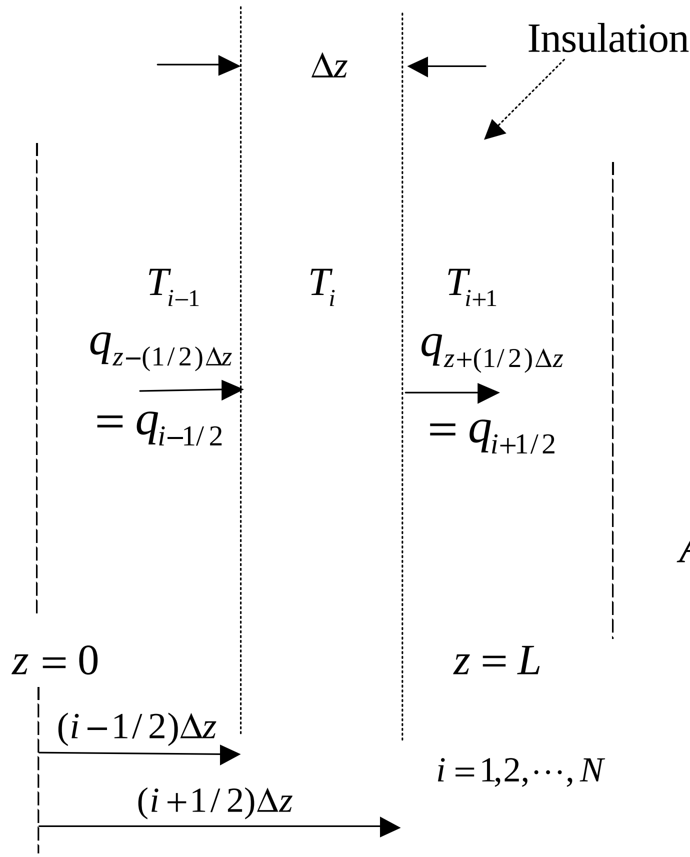


Energy Conservation Analysis by Finite Differences/Volumes



Energy Conservation Analysis by Finite Differences/Volumes



By energy conservation on a finite volume

accumulation = input - output

$$A \Delta z\, r\, C_p \frac{dT_i}{dt} = A(q_{i-1/2} - q_{i+1/2})$$

By a FD approximation to Fourier's first law

$$q_{i+1/2} = -k \frac{(T_{i+1} - T_i)}{\Delta z} \qquad q_{i-1/2} = -k \frac{(T_i - T_{i-1})}{\Delta z}$$

$$A \Delta z\, r\, C_p \frac{dT_i}{dt} = A\left[k\left(\frac{T_{i-1} - T_i}{\Delta z}\right) - k\left(\frac{T_i - T_{i+1}}{\Delta z}\right)\right]$$

$$\frac{dT_i}{dt} = \frac{\dfrac{k}{r C_p}\left[\left(\dfrac{T_{i-1} - T_i}{\Delta z}\right) - \left(\dfrac{T_i - T_{i+1}}{\Delta z}\right)\right]}{\Delta z}$$

$$\frac{dT_i}{dt} = a\left(\frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta z^2}\right)$$

For $\Delta z \to 0$, Fourier's second law: $\dfrac{\partial T}{\partial t} = a \dfrac{\partial^2 T}{\partial z^2}$

One IC, 2 BCs

Summary of Mathematical Model for Radiat ion Heating

$$-k\frac{\partial u(0,t)}{\partial z} = s\,(au_a^4 - eu^4(0,t))$$

Nonlinear radiation BC

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial z^2}$$

$$L_s r_s C_{ps}\frac{du_s}{dt} = -k\frac{\partial u(L,t)}{\partial z}$$

ODE/Neumam BC

$z = 0$ $\qquad$ $z = L$

u_a ambient temperature(K)

s Stefan - Boltzmann constant ($= 5.67\text{x}10^{-8}\,\frac{J}{s\cdot m^2\cdot K^4}$)

a absorptivi ty

e emissivity

k insulation thermal conductivi ty ($\frac{J\cdot m}{s\cdot m^2\cdot K}$)

a insulation thermal diffusivit y ($\frac{cm^2}{s}$)

L insulation thickness (m)

L_s steel half thickness (m)

r_s steel density ($\frac{\text{kg}}{\text{m}^3}$)

C_{ps} steel specific heat ($\frac{\text{J}}{\text{kg}\cdot\text{K}}$)

Mathematical Model for the Heating of Insulated Steel

The basic energy balance is Fourier's second law

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial z^2} \tag{1}$$

The initial and boundary conditions are:

$$u(z, 0) = u_o \tag{2}$$

$$-k\frac{\partial u(0,t)}{\partial z} = \sigma(a u_a^4 - e u^4(0,t)) \tag{3}$$

$$L_s \rho_s C_{ps} \frac{du_s}{dt} = -k\frac{\partial u(L,t)}{\partial z} = 0 \tag{4}$$

where, in SI (MKS) units:

u	insulation temperature K
t	time s
z	position in the insulation m
u_a	ambient (flame) temperature K
α	insulation thermal diffusivity $\frac{m^2}{s}$
k	insulation thermal conductivity $\frac{J \cdot m}{s \cdot m^2 \cdot K}$
σ	Stefan-Boltzmann constant $= 5.67x10^{-8} \frac{J}{s \cdot m^2 \cdot K^4}$
L	insulation half thickness m
a	absorptivity
e	emissivity
u_s	steel temperature K
L_s	steel half thickness m
ρ_s	steel density $\frac{kg}{m^3}$
C_{ps}	steel specific heat $\frac{J}{kg \cdot K}$

To facilitate the analysis, dimensionless independent variables are defined as

$$z' = z/L, t' = t\alpha/L^2 \tag{5)(6}$$

Substitution of eqs. (5) and (6) in eq. (1)

$$\frac{\partial u}{\partial (L^2 t'/\alpha)} = \alpha \frac{\partial^2 u}{\partial (L^2 z')^2}$$

or

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial z^2} \tag{7}$$

where the prime ($'$) has been dropped and it is understood that we are working with the dimensionless independent variables of eqs. (5) and (6).

Eqs. (2) to (4) transform to (again, with the prime finally dropped):

$$u(z,0) = u_o \tag{8}$$

$$-\frac{k}{L}\frac{\partial u(0,t)}{\partial z} = \sigma(a u_a^4 - e u^4(0,t)) \tag{9}$$

$$L_s \rho_s C_{ps} \frac{du_s}{d(L^2 t'/\alpha)} = -\frac{k}{L}\frac{\partial u(1,t)}{\partial z'} = 0$$

or

$$\frac{du_s}{dt} = -\left(\frac{kL}{\alpha}\right)\left(\frac{1}{L_s \rho_s C_{ps}}\right)\frac{\partial u(1,t)}{\partial z} = 0 \tag{10}$$

Eqs. (7) to (10) are the model equations programmed in the MOL code.

The initial parameters are

k	$1.0 \; \dfrac{J \cdot m}{s \cdot m^2 \cdot K}$
σ	$5.67x10^{-8} \dfrac{J}{s \cdot m^2 \cdot K^4}$
α	$1.0x10^{-6} m^2/s$
L	$0.1m$
a	1.0
e	1.0
u_a	$2500K$
u_o	$25C$
ρ_s	$7800.0 \dfrac{kg}{m^3}$
C_{ps}	$435.0 \dfrac{J}{kg \cdot K}$
L_s	$0.025m$

For the MOL solution, we use $n = 21, t_f$ (dimensionless) $= 0.5$ and output $u(0,t)$, $u(1/2,t), u(1,t)$ vs t (in minutes; note t (minutes) $= t$ (dimensionless) $L^2/\alpha/60$). In particular, we observe $u(1,t)$ as a function of t.

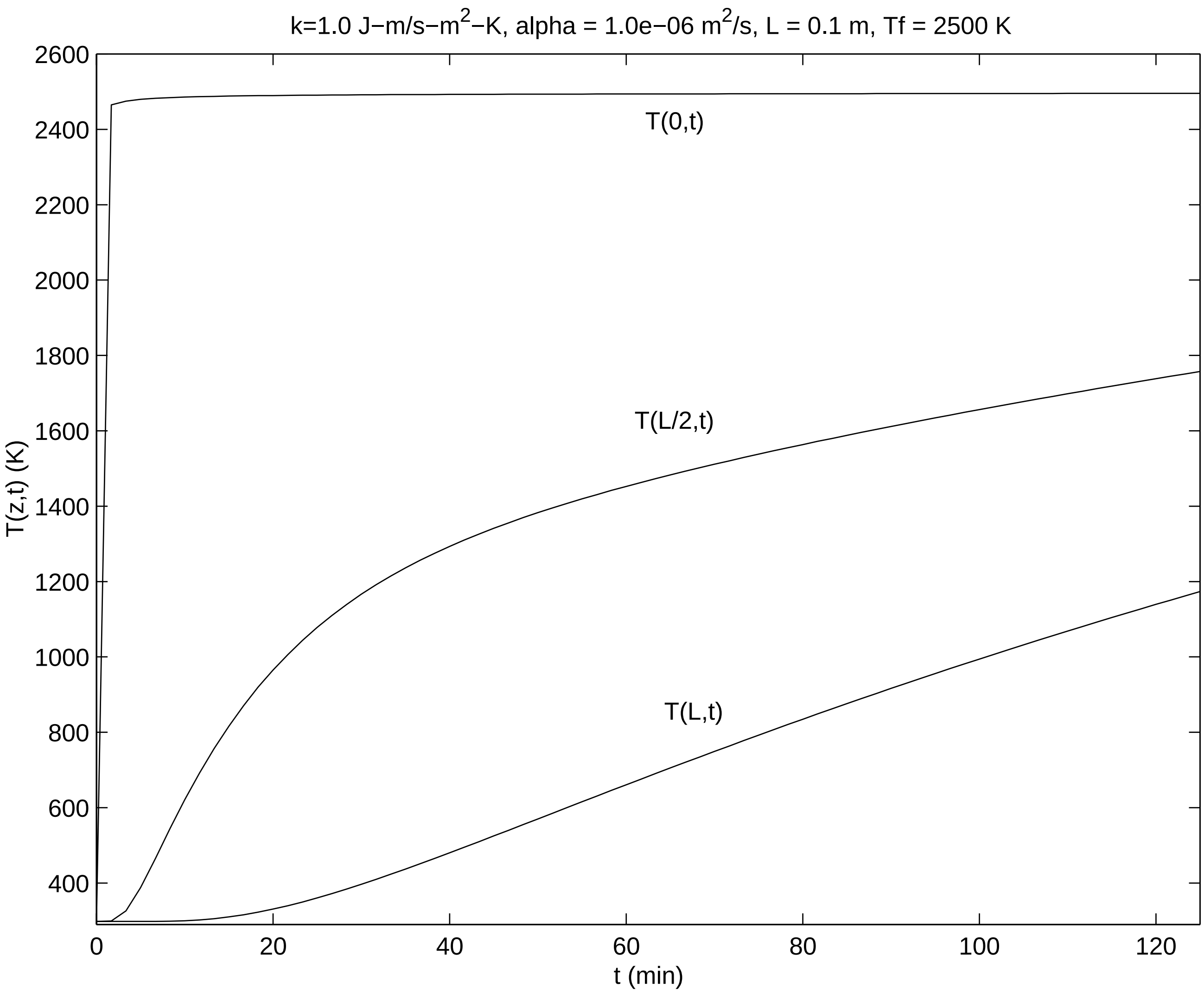
A Few Case Studies with the Mathematical Model

$$\text{Case 1}: \mathrm{k} = 1.0, a = 1.0\text{x}10^{-6}, L = 0.1$$

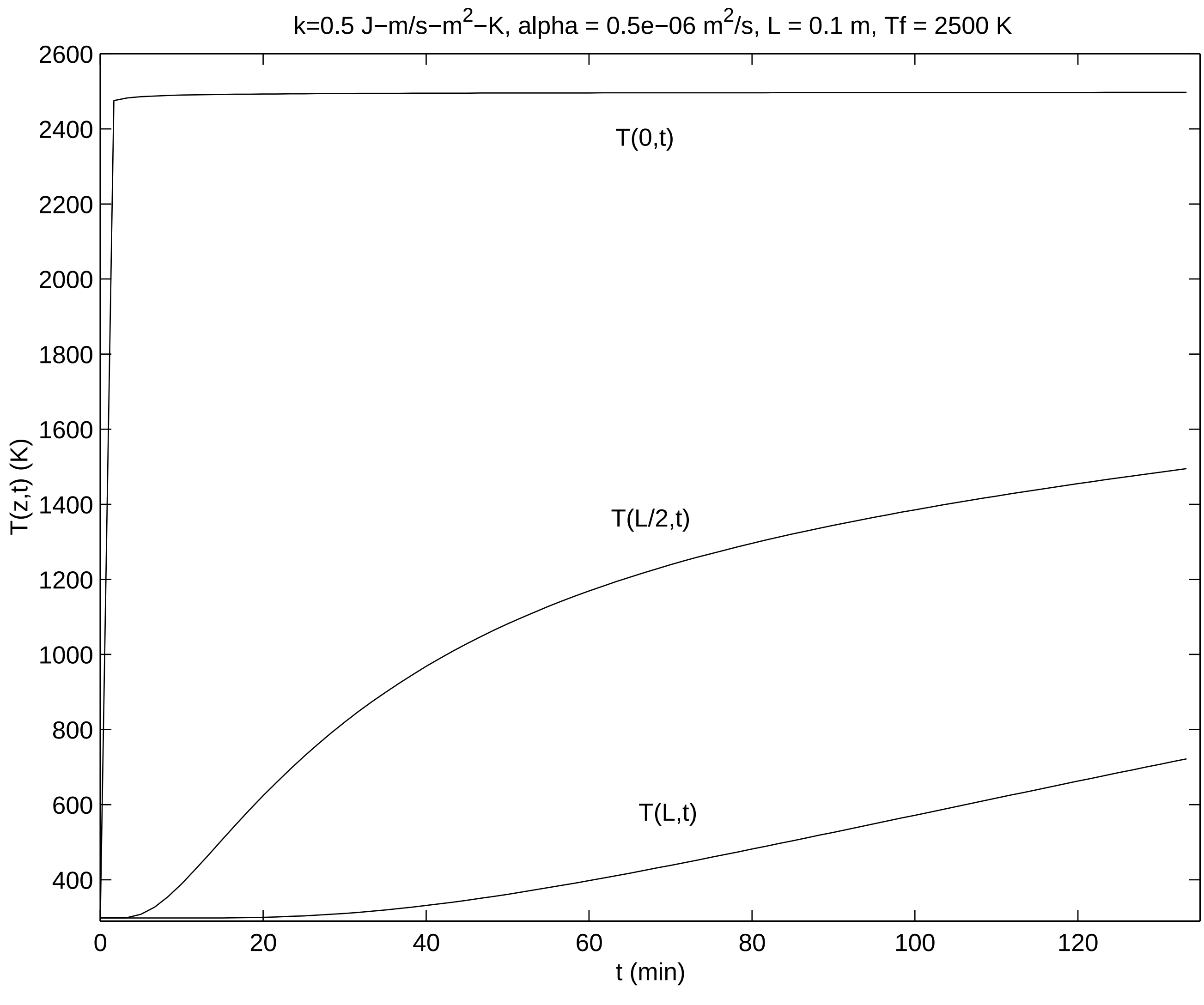
$$\text{Case 2}: \mathrm{k} = 0.5, a = 0.5\text{x}10^{-6}, L = 0.1$$

$$\text{Case 3}: \mathrm{k} = 1.0, a = 1.0\text{x}10^{-6}, L = 0.2$$

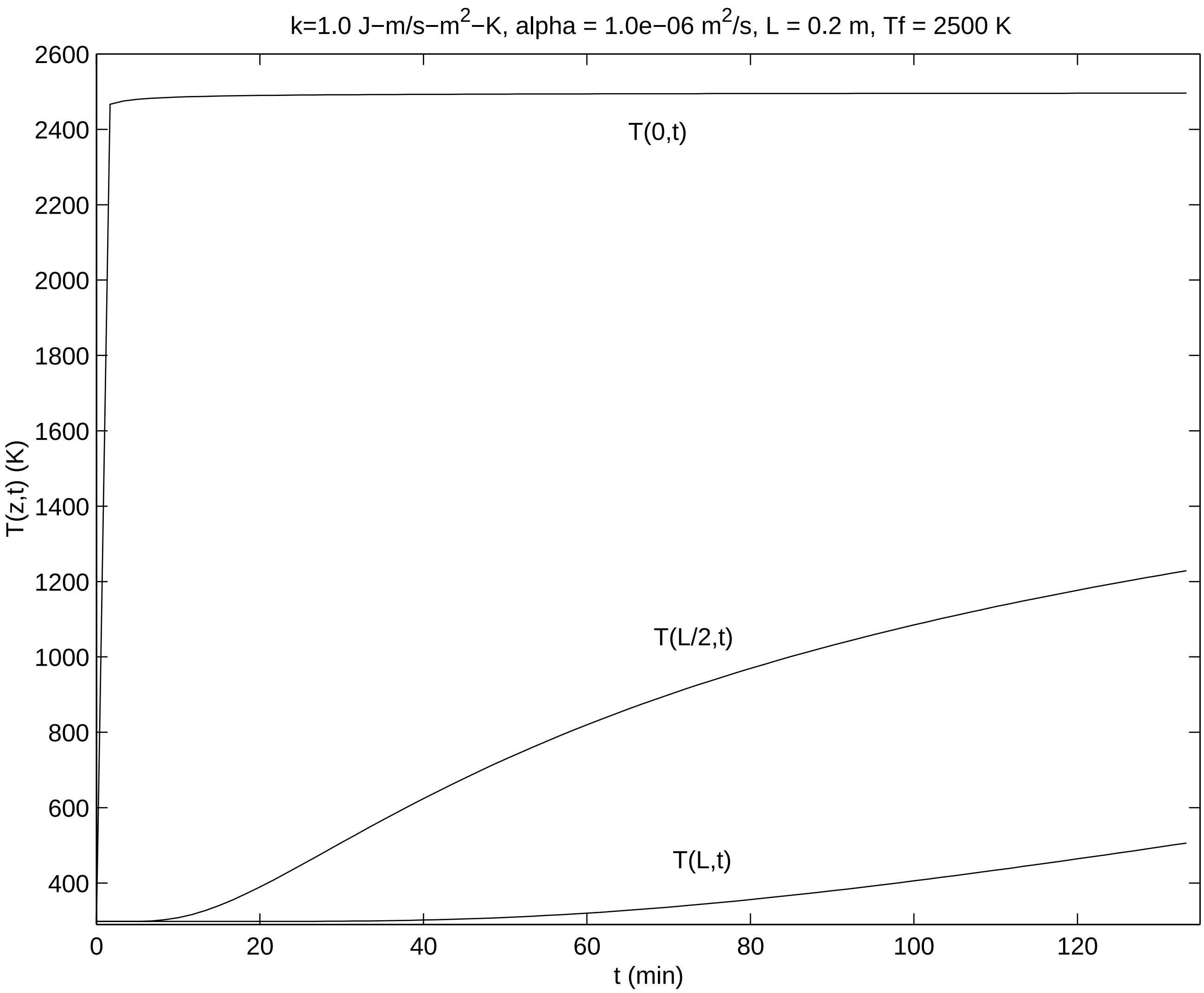
$$\text{Case 4}: \mathrm{k} = 2.0, a = 2.0\text{x}10^{-6}, L = 0.1$$



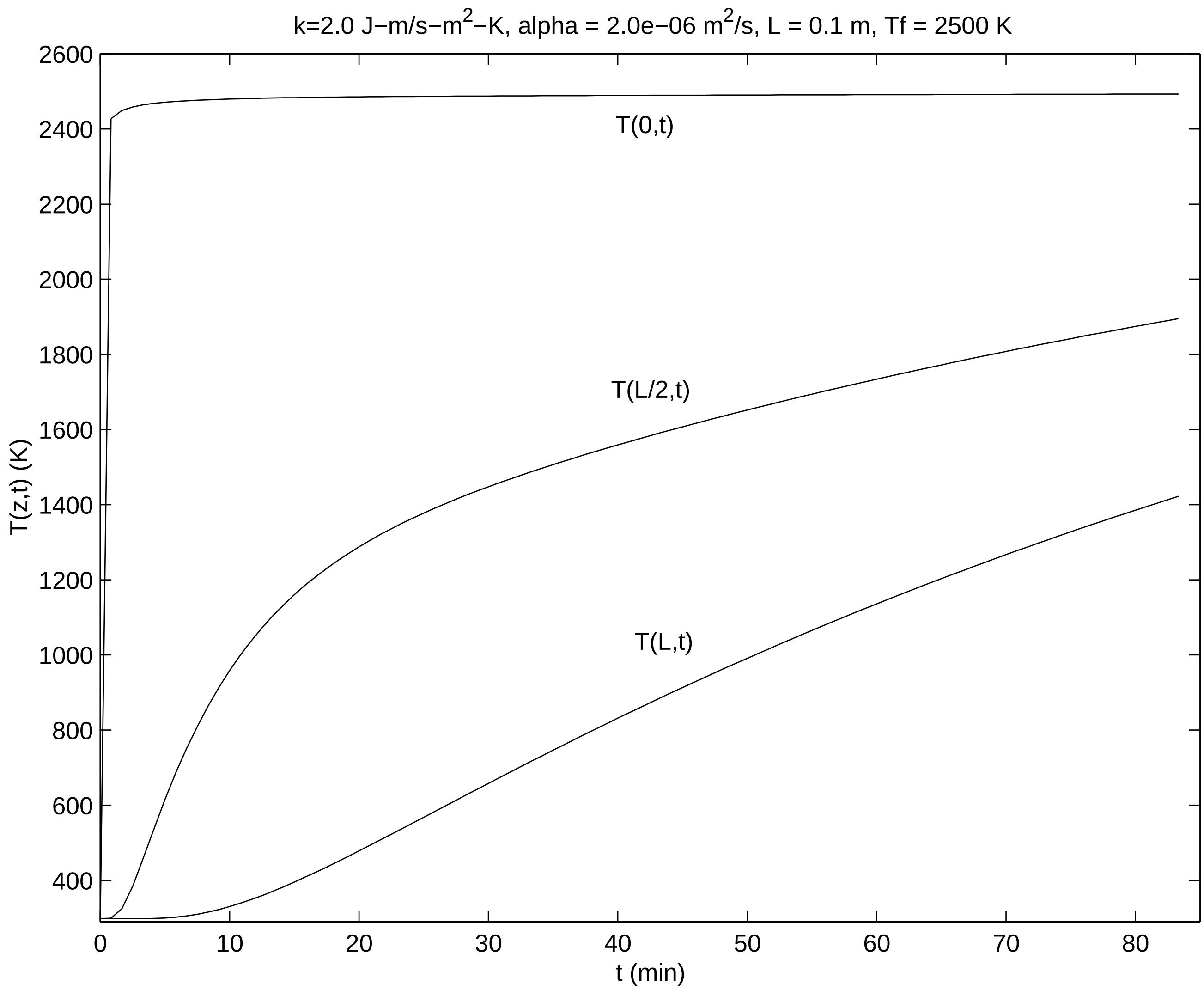
k=1.0 J−m/s−m2−K, alpha = 1.0e−06 m2/s, L = 0.1 m, Tf = 2500 K
T(0,t)
T(L/2,t)
T(L,t)
T(z,t) (K)
t (min)
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
0
20
40
60
80
100
120



k=0.5 J−m/s−m2−K, alpha = 0.5e−06 m2/s, L = 0.1 m, Tf = 2500 K
T(0,t)
T(L/2,t)
T(L,t)
T(z,t) (K)
t (min)
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
0
20
40
60
80
100
120



k=1.0 J−m/s−m2−K, alpha = 1.0e−06 m2/s, L = 0.2 m, Tf = 2500 K
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
T(z,t) (K)
T(0,t)
T(L/2,t)
T(L,t)
0
20
40
60
80
100
120
t (min)


k=2.0 J−m/s−m^2−K, alpha = 2.0e−06 m^2/s, L = 0.1 m, Tf = 2500 K
T(0,t)
T(L/2,t)
T(L,t)
T(z,t) (K)
t (min)
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
0
10
20
30
40
50
60
70
80