Learning FEMLAB/Matlab for Chemical Engineering
– Unsteady State Heat Conduction In A One-Dimensional Slab

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Discussion and Conclusions

Equations (1) has been solved using (a) finite differences in a spreadsheet using Excel (3), (b) the method of lines (4, 5) and (c) exactly (6). Table 1 compares the solutions at t = 0.12 and at various values of η. The FEMLAB solution was tabulated by reading point values on a graph of the solution curve. FEMLAB allows the user an ability to solve PDEs with an accuracy similar to other methods. Most important, FEMLAB has considerable capabilities to solve much more difficult problems.

The Model System

The parabolic differential equation

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial h^2} \qquad (1)$$

where

$$q = \frac{T_1 - T}{T_1 - T_0} \quad , \quad h = \frac{x}{b} \quad , \quad t = \frac{at}{b^2}$$

and

x	=	distance from center to a point in slab in m
b	=	distance from boundary to center of the slab in m
a	=	thermal diffusivity in m^2/s (k/ r C_p)
t	=	time in s
T_1	=	temperature at boundary
T_0	=	temperature in slab at t = 0
T	=	temperature in slab at time t

with boundary conditions

At t = 0, T = T_0 at all x
At t $\geq$ 0, T = T_1 at x = $\pm$ b

has been solved both analytically and numerically in a number of ways. If $T_1 < T_0$ then cooling takes place. If $T_1 > T_0$ then heating takes place (Fig. 1)

The FEMLAB Solution (1)

FEMLAB is a windows-based system built on Matlab (2) and is designed to numerically solve partial differential equations (PDE's) utilizing a finite element approach. It has a large number of built in equations that can be modified by the user. Applications in chemical engineering include momentum transport, energy transport and mass transport. Significantly, FEMLAB can solve the applicable equations simultaneously. A drawing facility allows the user to specify arbitrary geometries. Its output is generally graphical although numerical output is also available.

The following steps in FEMLAB are used in setting up and solving the above PDE for cooling (T_0 = 100, T_1 = 0)

Select:

1D (one dimensional)
  Chemical Engineering Module
    Cartesian Coordinates
      Energy Balance
        Heat Transfer
          Time Dependent

Draw Mode

The simple geometry indicated Fig (1) is entered with b = ±1

Boundary Mode

The temperature at the boundaries is entered, T_1 = 0 and the physical properties for r , C_p and k.

Subdomain Mode

The initial value T_0 = 100 is entered.

Mesh Mode

The mesh desired for the finite elements is specified. After the initial mesh is set, the mesh is refined twice.

Solve Mode

The values of t at which the solution is desired are entered, from 0 to 1 in increments of 0.01.

Post Mode

The temperature profile at t = 0.12 is selected. The profile can be zoomed in to get a better reading which may be done by pointing the mouse and clicking on the profile. The temperature is displayed in the lower left window.

An animation of the solution (temperature profiles at various values of t) can also be specified.

FEMLAB screens used in the problem can be found at the end of this document.

References

1. FEMLAB Chemical Engineering Module, COMSOL AB Version 2.3 http://www.comsol.com

2. The MathWorks, Inc Natick, MA http://www.mathworks.com

3. Rosen, E. M., "Reprise: Solving Partial Differential Equations Using Excel 2000" *CACHE News*, No 51 Fall 2000

4. Cutlip, M. B. and M. Shacham, "The Numerical Method of Lines for Partial Differential Equations", *CACHE News*, Fall 1998 p 18

5. Taylor, R. , Engineering Computing with maple: Solution of PDEs via the Method of Lines, CACHE News, No 49 Fall 1999 p. 5

6. Boelter, L.M. K., Cherry, V. H., Johnson, H. A. and Marteinelli, R. C., Heat Transfer Notes, University of California Press, Berkeley (1948) p V-12-b

Acknowledgements

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Figure 1. One Dimensional Slab

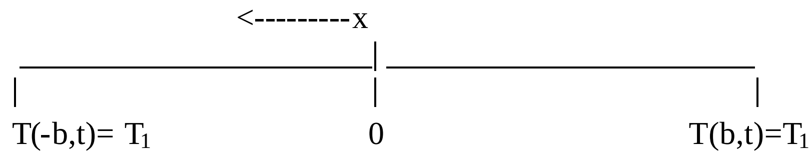
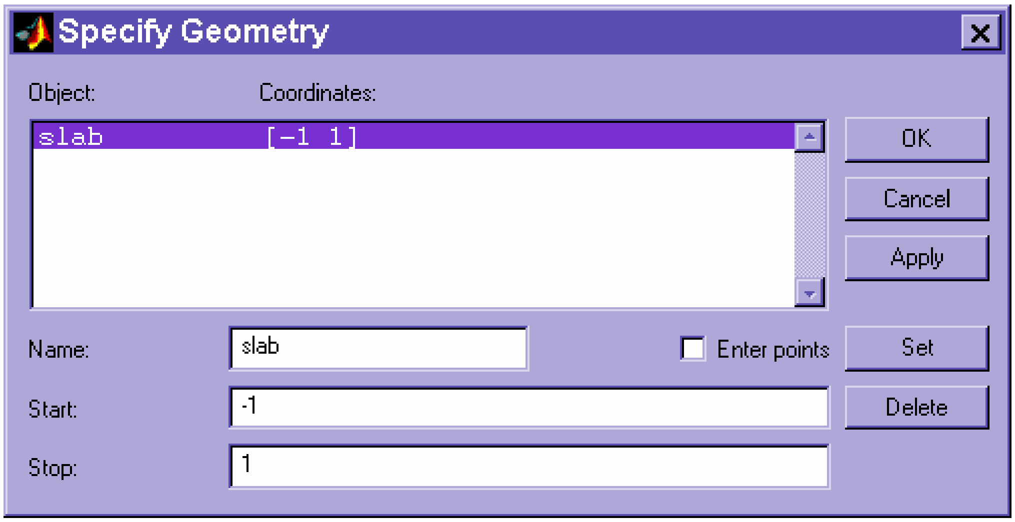


Table 1.

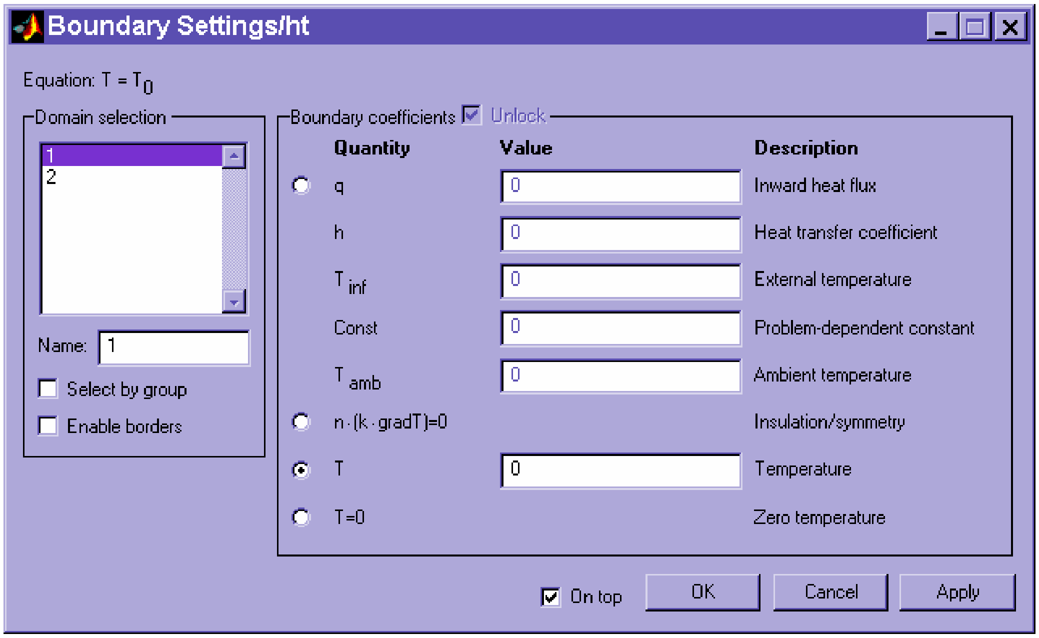
Comparison of Methods at t = 0.12 (Time = 6000 sec., a = 0.00002, b = 1)
T_0 = 100, T_1 = 0

h	Finite Differences (3)	Method of Lines(4)	FEMLAB Finite Elements (1)	Exact (6)
0	91.72	91.73	91.84	91.75
0.2	88.29	88.31	88.36	88.32
0.4	77.49	77.50	77.47	77.51
0.6	58.47	58.47	58.35	58.47
0.8	31.68	31.67	31.62	31.67
1.0	0.	0.	0.	0.

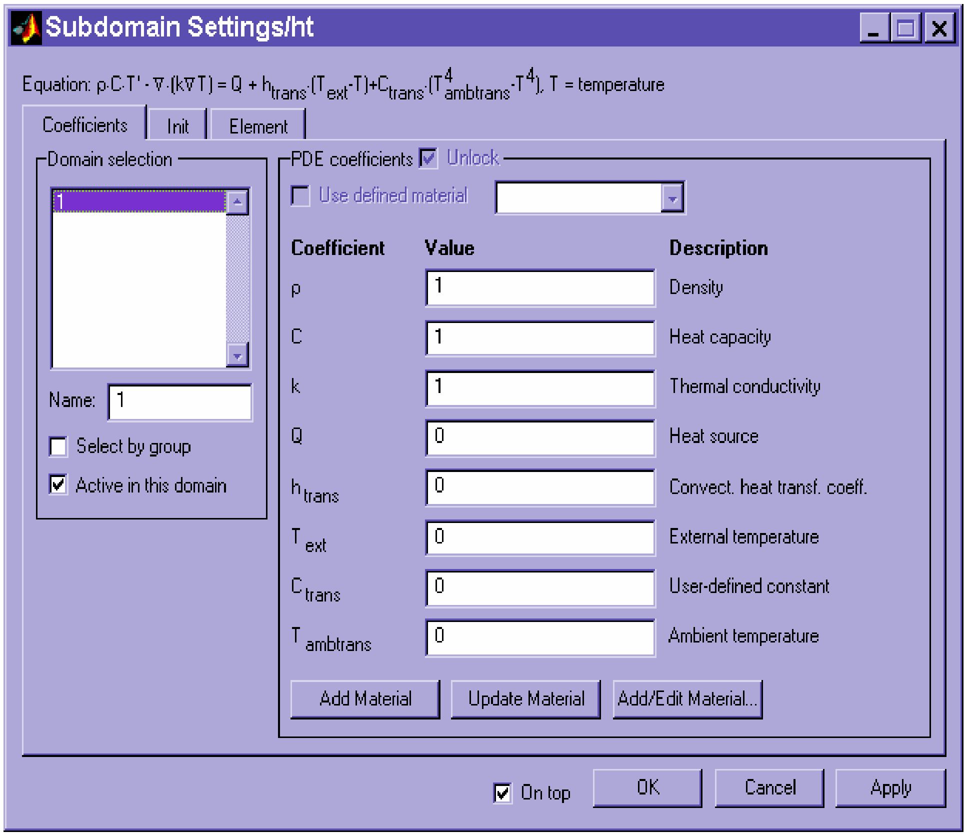
Draw Mode


Specify Geometry
Object:
Coordinates:
slab
[-1 1]
OK
Cancel
Apply
Name:
slab
Enter points
Set
Start:
-1
Delete
Stop:
1

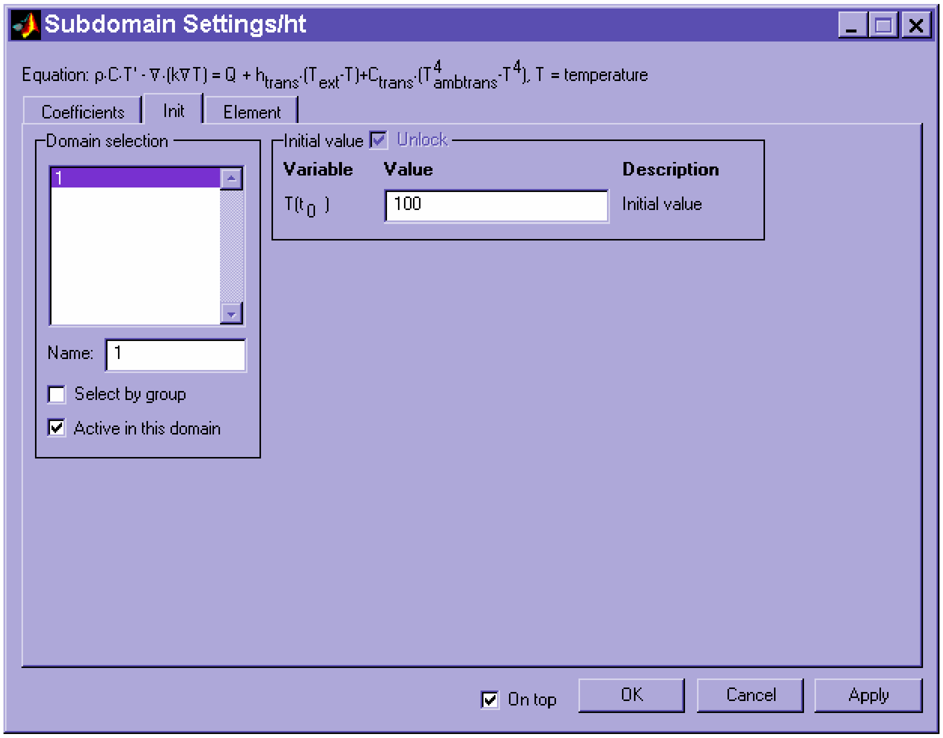
Boundary Mode



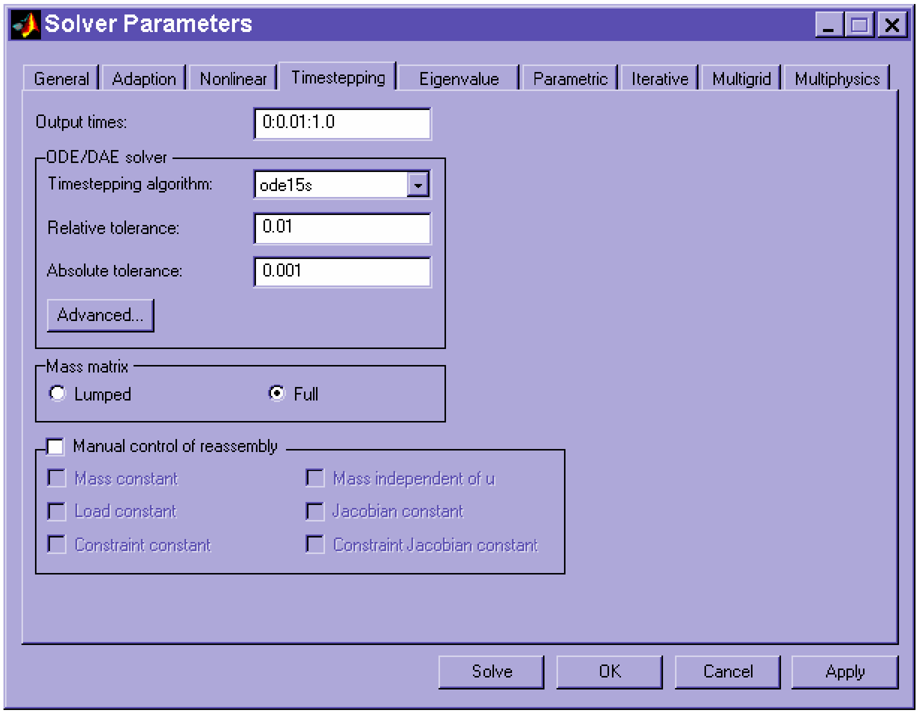
Boundary Mode – Physical Properties



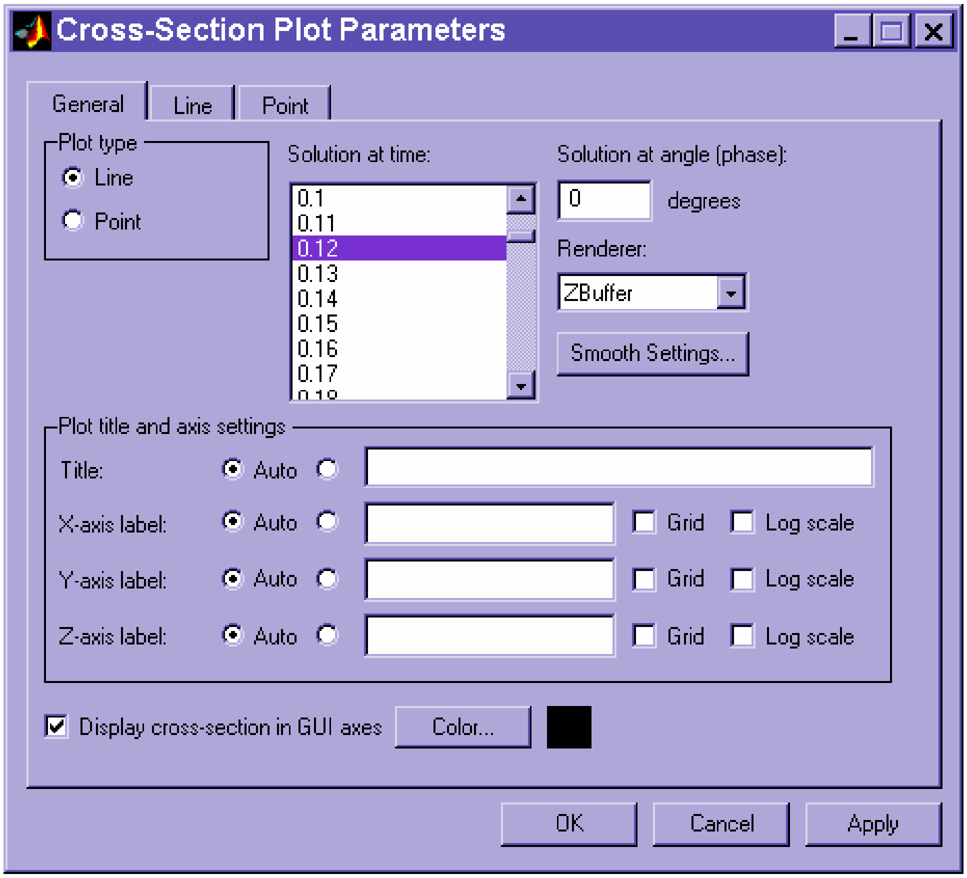
Boundary Mode – Initial Value



Solve Mode


Solver Parameters
General
Adaption
Nonlinear
Timestepping
Eigenvalue
Parametric
Iterative
Multigrid
Multiphysics
Output times:
0:0.01:1.0
ODE/DAE solver
Timestepping algorithm:
ode15s
Relative tolerance:
0.01
Absolute tolerance:
0.001
Advanced...
Mass matrix
Lumped
Full
Manual control of reassembly
Mass constant
Mass independent of u
Load constant
Jacobian constant
Constraint constant
Constraint Jacobian constant
Solve
OK
Cancel
Apply

Post Mode – Selection of Tau



Solution at Tau = 0.12