

Learning FEMLAB/Matlab for Chemical Engineering – Unsteady State Heat Conduction In A One-Dimensional Slab

Edward M. Rosen - EMR Technology Group

Discussion and Conclusions

Equations (1) has been solved using (a) finite differences in a spreadsheet using Excel (3), (b) the method of lines (4, 5) and (c) exactly (6). Table 1 compares the solutions at $t = 0.12$ and at various values of η . The FEMLAB solution was tabulated by reading point values on a graph of the solution curve. FEMLAB allows the user an ability to solve PDEs with an accuracy similar to other methods. Most important, FEMLAB has considerable capabilities to solve much more difficult problems.

The Model System

The parabolic differential equation

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial h^2} \quad (1)$$

where

$$q = \frac{T_1 - T}{T_1 - T_0}, \quad h = \frac{x}{b}, \quad t = \frac{at}{b^2}$$

and

- x = distance from center to a point in slab in m
- b = distance from boundary to center of the slab in m
- a = thermal diffusivity in m^2/s ($k/ \rho C_p$)
- t = time in s
- T_1 = temperature at boundary
- T_0 = temperature in slab at $t = 0$
- T = temperature in slab at time t

with boundary conditions

- At $t = 0$, $T = T_0$ at all x
- At $t \geq 0$, $T = T_1$ at $x = \pm b$

has been solved both analytically and numerically in a number of ways. If $T_1 < T_0$ then cooling takes place. If $T_1 > T_0$ then heating takes place (Fig. 1)

The FEMLAB Solution (1)

FEMLAB is a windows-based system built on Matlab (2) and is designed to numerically solve partial differential equations (PDE's) utilizing a finite element approach. It has a large number of built in equations that can be modified by the user. Applications in chemical engineering include momentum transport, energy transport and mass transport. Significantly, FEMLAB can solve the applicable equations simultaneously. A drawing facility allows the user to specify arbitrary geometries. Its output is generally graphical although numerical output is also available.

The following steps in FEMLAB are used in setting up and solving the above PDE for cooling ($T_0 = 100$, $T_1 = 0$)

Select:

1D (one dimensional)
Chemical Engineering Module
Cartesian Coordinates
Energy Balance
Heat Transfer
Time Dependent

Draw Mode

The simple geometry indicated Fig (1) is entered with $b = \pm 1$

Boundary Mode

The temperature at the boundaries is entered, $T_1 = 0$ and the physical properties for r , C_p and k .

Subdomain Mode

The initial value $T_0 = 100$ is entered.

Mesh Mode

The mesh desired for the finite elements is specified. After the initial mesh is set, the mesh is refined twice.

Solve Mode

The values of t at which the solution is desired are entered, from 0 to 1 in increments of 0.01.

Post Mode

The temperature profile at $t = 0.12$ is selected. The profile can be zoomed in to get a better reading which may be done by pointing the mouse and clicking on the profile. The temperature is displayed in the lower left window.

An animation of the solution (temperature profiles at various values of t) can also be specified.

FEMLAB screens used in the problem can be found at the end of this document.

References

1. FEMLAB Chemical Engineering Module, COMSOL AB Version 2.3
<http://www.comsol.com>
2. The MathWorks, Inc Natick, MA <http://www.mathworks.com>
3. Rosen, E. M., "Reprise: Solving Partial Differential Equations Using Excel 2000"
CACHE News, No 51 Fall 2000
4. Cutlip, M. B. and M. Shacham, "The Numerical Method of Lines for Partial Differential Equations", *CACHE News*, Fall 1998 p 18
5. Taylor, R. , Engineering Computing with maple: Solution of PDEs via the Method of Lines, *CACHE News*, No 49 Fall 1999 p. 5
6. Boelter, L.M. K., Cherry, V. H., Johnson, H. A. and Marteinelli, R. C., Heat Transfer Notes, University of California Press, Berkeley (1948) p V-12-b

Acknowledgements

This work was done during a trial period for FEMLAB/Matlab. The author would like to thank The MathWorks, Inc. and Comsol, Inc.

Peter R. Rony's comments have been incorporated into this paper. He supplied the Power Point copies of the FEMLAB screens as well.

Figure 1. One Dimensional Slab

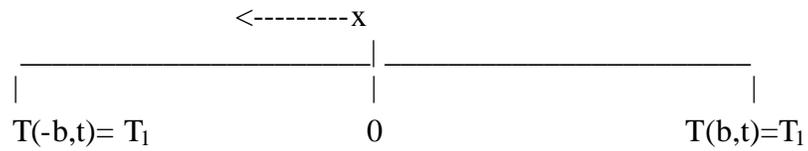
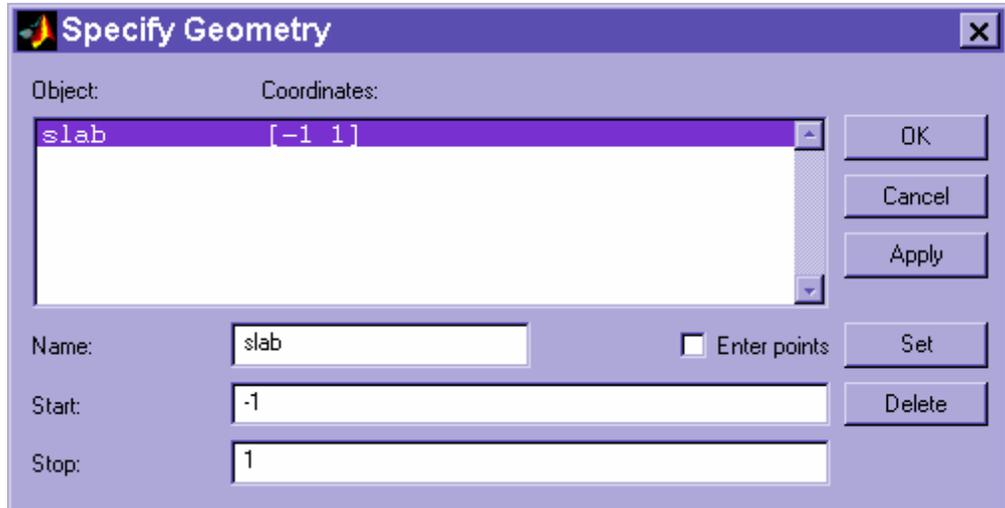


Table 1.

Comparison of Methods at $t = 0.12$ (Time = 6000 sec., $a = 0.00002$, $b = 1$)
 $T_0 = 100$, $T_1 = 0$

h	Finite Differences (3)	Method of Lines(4)	FEMLAB Finite Elements (1)	Exact (6)
0	91.72	91.73	91.84	91.75
0.2	88.29	88.31	88.36	88.32
0.4	77.49	77.50	77.47	77.51
0.6	58.47	58.47	58.35	58.47
0.8	31.68	31.67	31.62	31.67
1.0	0.	0.	0.	0.

Draw Mode



The image shows a software dialog box titled "Specify Geometry". It has a purple title bar with a close button (X) in the top right corner. The dialog is divided into several sections:

- Object/Coordinates Table:** A table with two columns, "Object" and "Coordinates". The first row contains the text "slab" and the coordinates "[-1 1]".
- Name:** A text input field containing the word "slab".
- Start:** A text input field containing the value "-1".
- Stop:** A text input field containing the value "1".
- Buttons:** On the right side, there are five buttons: "OK", "Cancel", "Apply", "Set", and "Delete".
- Enter points:** A checkbox labeled "Enter points" is located to the right of the "Name" field and is currently unchecked.

Boundary Mode

Boundary Settings/ht

Equation: $T = T_0$

Domain selection

1
2

Name: 1

Select by group
 Enable borders

Boundary coefficients Unlock

Quantity	Value	Description
<input type="radio"/> q	0	Inward heat flux
h	0	Heat transfer coefficient
T_{inf}	0	External temperature
Const	0	Problem-dependent constant
T_{amb}	0	Ambient temperature
<input type="radio"/> $n \cdot (k \cdot \text{grad}T) = 0$		Insulation/symmetry
<input checked="" type="radio"/> T	0	Temperature
<input type="radio"/> T=0		Zero temperature

On top OK Cancel Apply

Boundary Mode – Physical Properties

Subdomain Settings/ht

Equation: $\rho \cdot C \cdot T' \cdot \nabla \cdot (k \nabla T) = Q + h_{\text{trans}} \cdot (T_{\text{ext}} - T) + C_{\text{trans}} \cdot (T_{\text{ambtrans}}^4 - T^4)$, T = temperature

Coefficients | Init | Element

Domain selection

1

Name: 1

Select by group

Active in this domain

PDE coefficients Unlock

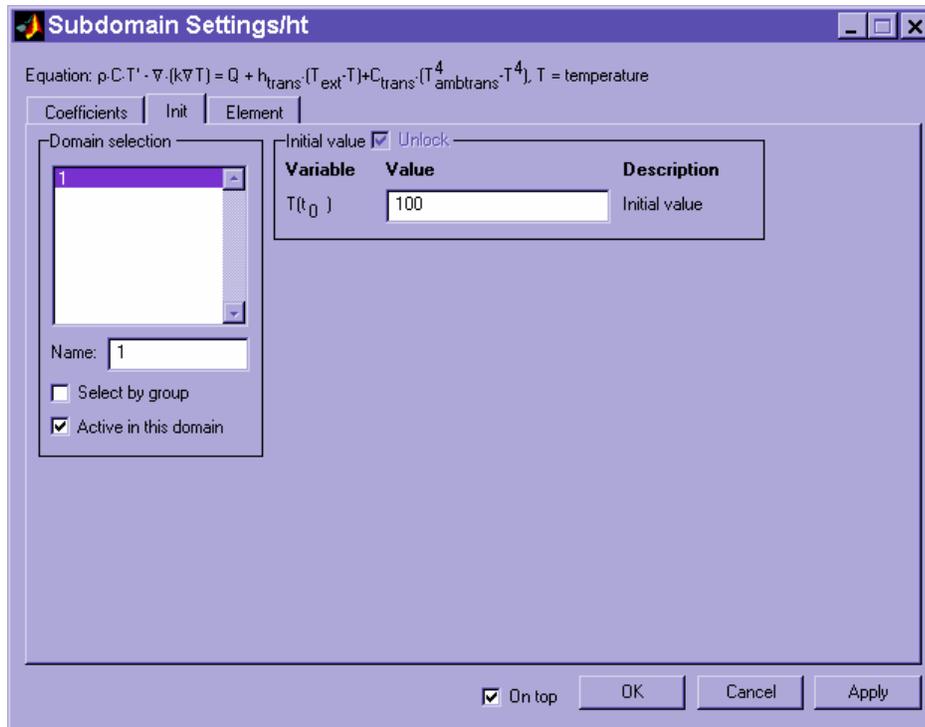
Use defined material

Coefficient	Value	Description
ρ	1	Density
C	1	Heat capacity
k	1	Thermal conductivity
Q	0	Heat source
h_{trans}	0	Convect. heat transf. coeff.
T_{ext}	0	External temperature
C_{trans}	0	User-defined constant
T_{ambtrans}	0	Ambient temperature

Add Material Update Material Add/Edit Material...

On top OK Cancel Apply

Boundary Mode – Initial Value



Solve Mode

Solver Parameters

General | Adaption | Nonlinear | **Timestepping** | Eigenvalue | Parametric | Iterative | Multigrid | Multiphysics

Output times: 0:0.01:1.0

ODE/DAE solver

Timestepping algorithm: ode15s

Relative tolerance: 0.01

Absolute tolerance: 0.001

Advanced...

Mass matrix

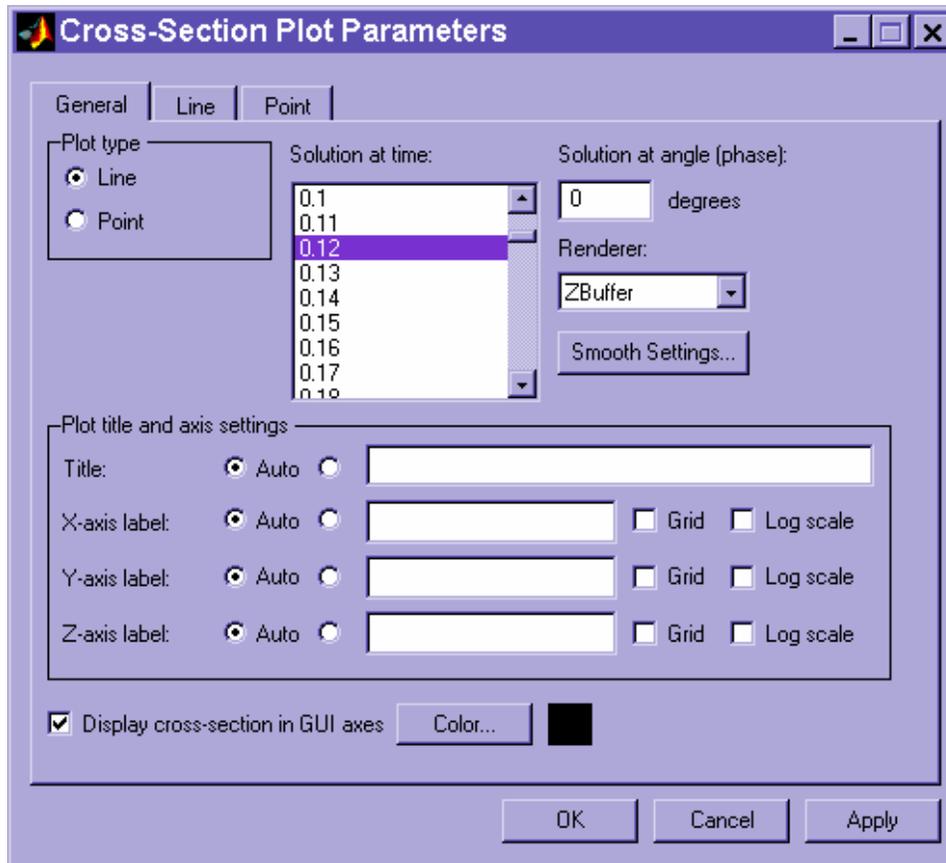
Lumped Full

Manual control of reassembly

<input type="checkbox"/> Mass constant	<input type="checkbox"/> Mass independent of u
<input type="checkbox"/> Load constant	<input type="checkbox"/> Jacobian constant
<input type="checkbox"/> Constraint constant	<input type="checkbox"/> Constraint Jacobian constant

Solve OK Cancel Apply

Post Mode – Selection of Tau



Solution at Tau = 0.12

