INTRODUCING DECISION MAKING UNDER UNCERTAINTY AND STRATEGIC CONSIDERATIONS IN ENGINEERING DESIGN

Georgia Kosmopoulou, Chintamani Jog, Margaret Freeman, and Dimitrios V. Papavassiliou
The University of Oklahoma • Norman, OK 73019

The material that is commonly taught in chemical engineering design for engineering economics could be described as "risk free," in the sense that the economic estimations presented to students appear to be free of financial uncertainty. The teaching of concepts in economics is usually focused on the treatment of the time value of money (*i.e.*, interest and inflation), the calculation of deterministic values for profitability criteria (*e.g.*, return on investment, net present worth), and the calculation of equipment cost and plant cost. Uncertainty is usually associated with limitations of the engineering models used to estimate the cost of the major pieces of equipment in the plant. For example, the students are taught that the models used to calculate the heat transfer area for a heat exchanger are based on semi-empirical correlations and, thus, the estimated cost of a heat exchanger might be inaccurate. The uncertainty about raw material and product prices, about the cost of energy and about labor cost, and the fact that the actual values might depend on factors that are outside of an engineer's control (*e.g.*, weather, natural disasters, international financial landscape) is not usually emphasized.

In fact, not only most textbooks in chemical engineering design but also most textbooks on engineering economics used within other engineering disciplines offer the same "risk free" content. Concepts such as cost estimation and profitability are, of course, quite important for quantifying the economic feasibility of an engineering project, but the availability of models that can handle financial risk, uncertainty, and decision making calls for an update of the instruction material. Recently, some effort has been placed on the introduction of risk analysis in chemical engineering design.[1] Uncertainty as well as other important concepts such as decision tree analysis and utility functions, however, have not been part of a typical undergraduate curriculum.

Lately, through collaboration between the University of Oklahoma Department of Chemical Engineering and Department of Economics, we have developed classroom games that demonstrate concepts such as strategic decision making, the winner's curse, and the utility function in Design I—a course that introduces engineering economics to chemical engineers who lack an extensive economics background. In this paper, we discuss the development of these games (or class experiments, as they would be called in the economics literature) and the educational objectives of each game. We also demonstrate the basic components of these games and we discuss the mechanics of carrying out experiments in the classroom. The concepts that are visited with the games can be used to quantify risk and facilitate decision making under uncertainty.

TAKING FINANCIAL UNCERTAINTY INTO ACCOUNT

Uncertainty and change are pervasive in the careers of new engineers, and mastering appropriate analysis techniques and tools will be greatly beneficial to graduates. A rather easy concept for the students to grasp is the incorporation of uncertainty in the decision-making process by maximizing expected profits. A good example for introducing expected profit to students is the drilling of an oil well (this is an example offered in detail as a case study in the textbook by Mansfield[2]) or the rolling

***Georgia Kosmopoulou** is the Edith Kinney Gaylord Presidential Professor at the Department of Economics at the University of Oklahoma. She received a B.S. from the University of Piraeus and her M.S. and Ph.D. from the University of Illinois at Urbana-Champaign. Her research interests are in industrial organization, auction theory, and mechanism design.*

***Chintamani Jog** is a Ph.D. student in the Department of Economics at the University of Oklahoma. He received his B.A. degree in economics from the University of Mumbai and M.A. in economics from Gokhale Institute of Politics and Economics, Pune.*

***Margaret Freeman** is a junior in chemical engineering at the University of Oklahoma. She is also pursuing a minor in entrepreneurship for engineering majors and is the recipient of an NSF-REU fellowship at Oklahoma.*

***Dimitrios Papavassiliou** is a Presidential Professor at the School of Chemical, Biological, and Materials Engineering at the University of Oklahoma. He received a Diploma from the Aristotle University of Thessaloniki, and M.S. and Ph.D. degrees from the University of Illinois at Urbana-Champaign. His research interests are in turbulent transport, transport in porous media, microfluidics, and computational transport. He has been teaching Design I during the past 10 years.*

out of a new product line. A company cannot be sure in advance whether it should be done and what the costs are going to be. Based on the best expert opinion that a company has, a probability density function can be obtained with discrete outcomes associated with possible profit. The expected profit is then given as

$$E(\pi) = \sum_{i=1}^{N} P_i \pi_i \qquad (1)$$

where N is the number of possible outcomes, P_i is the probability that outcome i will occur, and π_i is the profit for outcome i.

In the examples that follow, the choice of the values of the discrete probability function was made arbitrarily. In practice, however, one cannot generate the probability density function in a rigorous statistical manner, since one cannot be placed in the same business conditions and be faced with the same decision possibilities repeatedly. A particular business situation usually occurs once, thus, one cannot generate a sample of outcomes given the decisions made. The probability density function is usually generated after brainstorming and after consulting with experts having prior experience in similar situations. In the case of rolling out a new product one needs to use market analysis and surveys, in the case of pricing raw materials and products one needs to use forecasting techniques, and in the case of drilling a well one needs to rely on the opinion of geologists and geophysicists who are experienced in the interpretation of geological data (such as data obtained through seismic analysis or core analysis).

Example 1: Assume that if a new product (say raspberry-flavored Cola) is produced, there will be a 40% likelihood that it will not catch up in the market, 25% probability that it will get 1% of the competitors' market share, 20% probability that it will get 2% of that market, and 15% probability of a 3% additional market share. If the product is not rolled out, there will be no profit. Figure 1 is a decision tree presenting the payoffs arising from choices made by the decision maker and by chance outcomes. The expected profit from no change in the product line is zero while the expected profit from the introduction of a new product is

$$E(\pi) = 0.4(-20,000) + 0.25(50,000) + 0.20(100,000) + 0.15(150,000) = \$47,000.$$

If one makes decisions trying to maximize expected profit, then one should clearly decide to produce the new product, since the expected profit from that decision is higher than the expected profit from no change in production.

An example like this introduces students to a methodology for taking uncertainty into account, and provides the opportunity to discuss decision tree analysis (see Reference 2, or any other managerial economics textbook, for more on decision trees). The discussion of expected profit, however, also leads to the opportunity to discuss the utility functions as a way to quantify uncertainty and a way to incorporate the attitude of the decision maker towards risk. This discussion can start in the classroom by considering a case where the expected profit of two options is about the same, but one of the two is much more risky.

Example 2: A company can invest in a process that can yield a net present worth (NPW) of $1,000,000 with no risk, and a process that can have either a NPW of $2,150,000 with probability of 50% or a negative NPW of -$50,000 with a 50% probability. The expected NPW for the risky option is $1,050,000. Which option would the students pick?

In this example, the criterion of maximizing expected NPW in order to account for uncertainty conflicts with common sense. The students can see that there might be a company that cannot afford a 50% probability of losing $50,000, especially if this is a small company that could go out of business! The utility function can be used to quantify the attitude towards risk and to justify a decision that is clearly not based on expected profit maximization. But, what *is* a utility function?

It is a function that places a numerical value on happiness, or more specifically on the willingness to buy different goods or take different actions for companies and consumers! On Sept. 6, 2007, the student newspaper at the University of Oklahoma campus (*The Oklahoma Daily*) ran a half-page article entitled "Happiness Has a Personal Side" with pictures of 10 students and their responses to the question "What makes you happy?" Chocolate, shopping, family, and penguins were among the responses—answers that make good sense to students in the Design class. This article illustrated very nicely that happiness (and the utility function that attempts to quantify it) is subjective, and it was quite effective as a handout for relating the concept of the utility function to each student. A utility function has to be ordinal as it is difficult to quantify differences in happiness from different decisions and actions. The ordinal character of a utility allows people

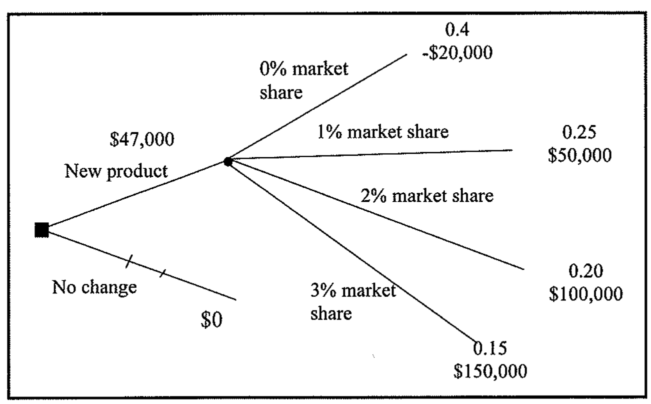


Figure 1. *Decision tree that graphically depicts the probable outcomes of rolling out a new product. The probability of each outcome is shown on the decision tree, as well as the profit or loss if the outcome occurs. The dollar value that appears on each branch is the expected profit that corresponds to the branch.*

to express their preferences between the no-risk option and the risky option in Example 2 and be consistent in choosing different courses of action.

How can a utility function be constructed? First, one can assign arbitrary values to the extremes of the possible profits.[2] For example, using the numbers offered in Example 1, we can say that the utility function has a value of zero for a loss of $20,000 and the value of 100 for a profit of $150,000. Any two arbitrarily chosen numbers would work, so long as the value of the utility function for the minimum profit is smaller than the value of the utility for the maximum profit. The value of the utility function at any intermediate profit π between the two extreme values π_{min} and π_{max} is found by determining the probability P for which the decision maker is indifferent between a risky option that includes the extremes and a safe option with profit π. In particular:

$$U(\pi) = P\,U(\pi_{min}) + (1-P)\,U(\pi_{max}) \qquad (2)$$

In other words, the person whose utility function is generated in this exercise is equally happy to take a safe bet with a return equal to π and a gamble with probability P of a return π_{min} and probability (1- P) of a return π_{max}. In this respect, the decision on whether to commit to one financial option or another can be based on *maximizing the expected utility*, calculated as

$$E(U) = \sum_{i=1}^{N} P_i\,U(\pi_i) \qquad (3)$$

where $U(\pi_i)$ is the value of the utility function for profit π_i.

To demonstrate to the students the utility function concept and to illustrate how a utility function can be generated, we have prepared a game that can be played by the students in class.

__Game 1:__ The game presents students with a series of two options, one of which is a gamble and the other a safe choice. In Figure 2, we are trying to determine the utility of

GAME 1

You need to make a series of choices between Options 1 and 2 and record your choices in the spreadsheet before rolling a fair 10-sided die. Consider your payoffs as the sum of the payoffs from each choice.

You need to choose between a "safe" bet, which is Option 2, and a "gamble" that can result in either a loss of $20,000 or a win of $150,000, based on the dice-rolling outcomes, which is Option 1.

• 1st choice:

Option 1: If the die comes up 1, you win $150,000, but if it comes up 2,3,..., 10 you lose 20,000.

Option 2: You win $0 no matter what the dice-rolling outcome is.

Make your choice between these two options and record your choice by typing 1 or 2 in the appropriate cell in the column titled "Record choice (1 or 2)" (column D in your Excel Sheet).

• 2nd choice

Option 1: If the die comes up 1, 2, you win $150,000, but if it comes up 3,..., 10 you lose 20,000.

Option 2: You win $0 no matter what the dice-rolling outcome is.

Again make your choice between these two options and record it by typing 1 or 2 in the appropriate cell in the column titled "Record choice (1 or 2)."

Continue up to the tenth choice having in mind that as you go down the list the probability of a favorable outcome in Option 1 increases by 1/10.

• Record the dice rolls in the column titled "Record dice roll (1-10)."

	B	C	D	E
1			Record choice	Record dice
2	Option 1	Option 2	(1 or 2)	roll (1-10)
3	Roll 1 to win $150,000, roll 2-10 to lose $20,000	0		
4	Roll 1,2 to win $150,000, roll 3-10 to lose $20,000	0		
5	Roll 1-3 to win $150,000, roll 4-10 to lose $20,000	0		
6	Roll 1-4 to win $150,000, roll 5-10 to lose $20,000	0		
7	Roll 1-5 to win $150,000, roll 6-10 to lose $20,000	0		
8	Roll 1-6 to win $150,000, roll 7-10 to lose $20,000	0		
9	Roll 1-7 to win $150,000, roll 8-10 to lose $20,000	0		
10	Roll 1-8 to win $150,000, roll 9,10 to lose $20,000	0		
11	Roll 1-9 to win $150,000, roll 10 to lose $20,000	0		
12	Any roll wins $150,000	0		
13				
14			Actual sum of return	
15			Expected sum of return	
16				
17				
18				

Game 1 / Game 2 / Game 3 / Game 4 / Game 5 / Utility Function / Retu

Figure 2. Example of the game that students play in order to determine the value of the utility function of a profit equal to zero (Option 2). Option 1 is a gamble between the two extreme values of the profit, taken from Example 2, with different probability of winning or losing, and thus different expected utility. The expected utility at which the player switches from the safe option to the gamble is the point of indifference and indicates the value of U(0).

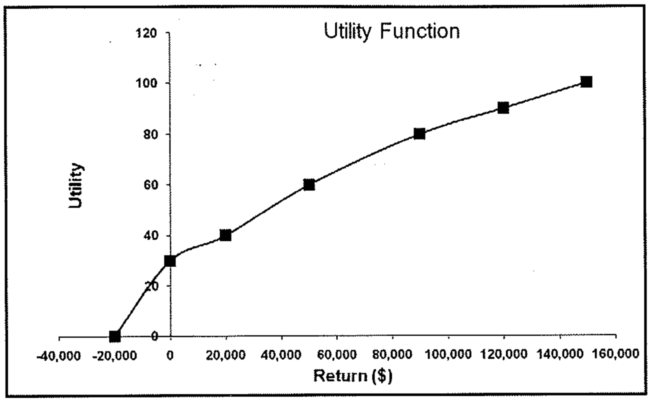


Figure 3. Typical utility function resulting from Game 1. This is the utility function of a risk-averse person, since the value of the utility increases at decreasing rate as the expected return increases.

zero profit by offering a set of choices between U(0) and a gamble between U(150,000) and U(-20,000). The players are asked to select their options and to input their selection in the spreadsheet.

The game is constructed using Microsoft Excel, which allows the instructor to lock certain cells of the spreadsheet and to pre-arrange the figures to present the data from specific parts of the spreadsheet. Student players cannot input numbers in places that can alter the structure of the game. The students play this game five times for five different safe profit values. When the game ends, each student's utility function for this example has been constructed, and some of the students can e-mail their spreadsheets to the instructor or place their utility function on a memory stick and show it to the rest of the class. At that point a discussion in class can be initiated on whether the person whose utility function is shown is a "risk-loving" or a "risk-averse" person. In addition, another discussion can be initiated based on the question of what the utility of a different amount than those chosen for the five games might be. Figure 3 is a typical outcome for the utility function from this game.

Having created the utility function, the students can return to Example 1 and apply the maximization of expected utility as a decision criterion. They can calculate the expected utility of rolling out a new product, compare it with that of not changing the production line, and make a choice. This will give students an immediate application of the concepts learned. More importantly, it becomes evident that this approach is bound to give different answers for different people, since it is the utility and not the profit that is maximized. Different attitudes towards risk are not captured by mere expected profit maximization, but they are taken into account when expected utility is maximized.

ATTITUDE TOWARDS RISK

The second classroom game builds on the first one. It is intended to be an application of the risk preferences seen earlier, introducing the ideas of "actual" and "expected" values.

GAME 2

Due to the EPA's concern over rising levels of greenhouse gasses, all chemical companies are required to reduce CO_2 emission by 15%. For the past few years, Independent Chemical, Inc., has subcontracted another company to process CO_2. The cost keeps rising, however, and Independent Chemical, Inc., is considering handling the CO_2 capturing in-house. Your consulting company is hired by Independent Chemicals, Inc. They want you to suggest which of the following technology options for CO_2 sequestration they should use:

Option 1: Build a bio-energy carbon storage plant next to their existing facility to capture and store CO_2.

Option 2: Use gas hydrate technology to transport CO_2 and store it in ocean.

Problem: Both technologies are not well tested, so there is no certain estimate of the profits, and if there are technical issues that will arise with the new processes, there might even be fines from the EPA.

Game Guidelines

• You need to make a choice between Options 1 and 2 and record your choice in the spreadsheet before rolling a fair 10-sided die that determines the outcome.

You need to choose between Option 1, which even in the case of failure can make money, and Option 2, which, in the case of failure, will result in a loss of $500,000.

• 1st choice:

Option 1: If the die comes up 1, you win $2,000,000, but if it comes up 2,3,..., 10 you win $1,000,000.
Option 2: If the die comes up 1, you win $3,850,000, but if it comes up 2,3,..., 10 you lose $500,000.

Make your choice between these two options and record your choice by typing 1 or 2 in the appropriate cell in the column titled "Record choice (1 or 2)" (column D in your Excel Sheet).

• Continue making choices up to the tenth set of options having in mind that as you go down the list the probability of the most favorable outcome in both options increases by 1/10.

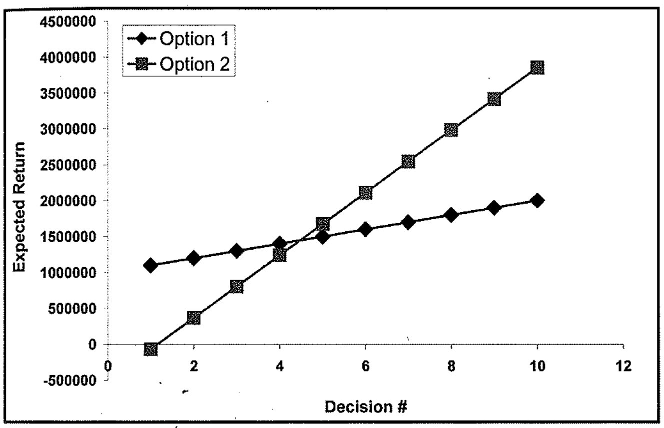
Game 2: The game presents students with a series of two options, each one with different expected profit and different level of risk. This game is also constructed using Microsoft Excel so that spreadsheet cells can be locked and figures can be generated based on the responses of the students. Figure 4 is a snapshot of the excel spreadsheet at the beginning of the game.

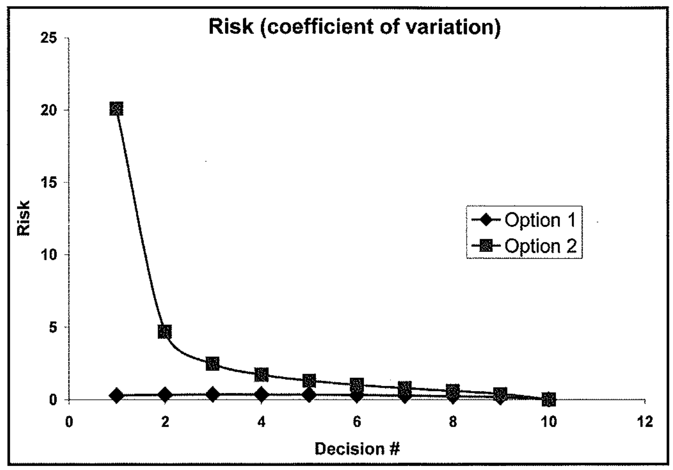
The expected return for each of the options in the game is different. Figure 5a is a plot of the expected profit for both of

Option 1	Option 2	Record choice (1 or 2)	Record dice roll (1-10)
Roll 1 to win $2 mil, roll 2-10 to win $1 mil	Roll 1 to win $3.85 mil, roll 2-10 to lose $0.5 mil		
Roll 1,2 to win $2 mil, roll 3-10 to win $1 mil	Roll 1,2 to win $3.85 mil, roll 3-10 to lose $0.5 mil		
Roll 1-3 to win $2 mil, roll 4-10 to win $1 mil	Roll 1-3 to win $3.85 mil, roll 4-10 to lose $0.5 mil		
Roll 1-4 to win $2 mil, roll 5-10 to win $1 mil	Roll 1-4 to win $3.85 mil, roll 5-10 to lose $0.5 mil		
Roll 1-5 to win $2 mil, roll 6-10 to win $1 mil	Roll 1-5 to win $3.85 mil, roll 6-10 to lose $0.5 mil		
Roll 1-6 to win $2 mil, roll 7-10 to win $1 mil	Roll 1-6 to win $3.85 mil, roll 7-10 to lose $0.5 mil		
Roll 1-7 to win $2 mil, roll 8-10 to win $1 mil	Roll 1-7 to win $3.85 mil, roll 8-10 to lose $0.5 mil		
Roll 1-8 to win $2 mil, roll 9,10 to win $1 mil	Roll 1-8 to win $3.85 mil, roll 9,10 to lose $0.5 mil		
Roll 1-9 to win $2 mil, roll 10 to win $1 mil	Roll 1-9 to win $3.85 mil, roll 9,10 to lose $0.5 mil		
Any roll wins $2 million	Any roll wins $3,850,000		
		Actual sum of return	
		Expected sum of return	

Return / Risk / Decision / Sheet2 / Sheet3

Figure 4. *Example of the game that students play in order to determine their attitude towards risk. Option 1 is a gamble that has a smaller risk, but it also has a smaller expected profit beyond the fifth row.*





Figures 5. *(a, left) Expected profit for options in Game 3; (b, right) corresponding estimate of risk for options in Game 3. The risk is always higher for Option 2, while the expected profit of Option 1 is higher for Option 1 until the fourth choice.*

the game options. One can clearly see that the expected return for Option 2 becomes higher than that for Option 1 after the fourth choice. As one can see in Figure 5b, however, the risk is higher for Option 2 every single time, until the tenth choice. Figure 5b is a plot of the risk for each game option expressed as the coefficient of variation (*i.e.*, the standard deviation normalized by the expected value).

If one were taking into account only expected returns, they should be choosing Option 1 for the first, second, third, and fourth choices, and then switch to Option 2. If one is risk-averse, they would stay with Option 1 even after the fourth choice, preferring to have a lower expected value but assuming a lower risk. The spreadsheet also allows the students to generate a figure called "Your decision," where they can see a plot of the expected value of the return for their choices and of the actual return as a result of the dice rolls. The thick solid line traces the choices made, and can indicate whether the player is risk-loving, risk-averse, or risk-neutral.

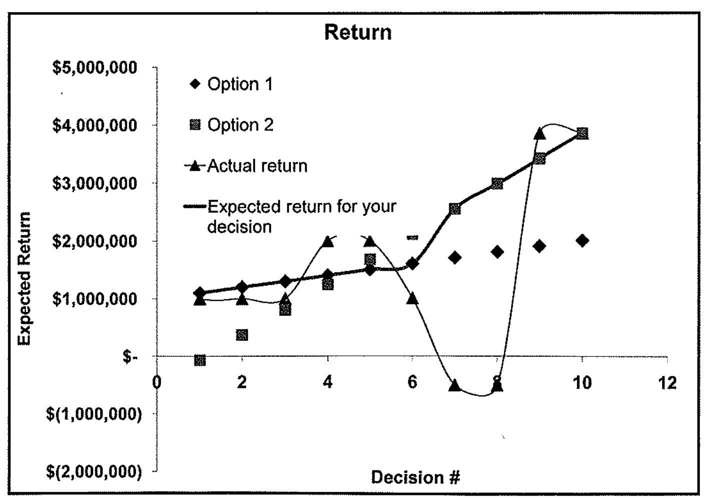


Figure 6. *Typical outcome of Game 3. The thick solid line indicates the expected profit based on the options chosen by this player. This is a risk-averse person, who chooses a lower expected profit with smaller risk. The triangles indicate the actual return based on the outcome of the particular dice roll.*

At the conclusion of the game, some of the students e-mail their "decision" figure to the instructor, in order to show it to the rest of the class (as in Figure 6, previous page). A discussion about whether this would be a curve characteristic of a "risk-loving" or a "risk-averse" person can be initiated. A risk-neutral person would try to maximize expected payoff as their only objective. Hence, the switch from Option 1 to Option 2 would occur at the fifth decision. Thus, for a potential risk lover, the switch should occur before the fifth decision and for a potential risk averter, same should be true beyond the fifth choice. The best part of this experiment is that students can see it for themselves merely by observing each other's diagrams and the decisions made.

DECISION MAKING IN COMPETITIVE SITUATIONS AND WINNER'S CURSE

In business, a strategic decision-making process is characterized by the firm's actions and counteractions leading to payoffs that vary with the outcome of the interaction process. Besides having uncertainty about the outcome of a process, there is also uncertainty about how the competitors will behave. The strategic interactions among competing firms can lead to changes in the production level that affect the technical and economic functions of a company. A simple game that can illustrate this concept, and is quite easy to do in class, is the following:

Game 3: The students are asked to write on a piece of paper a number between 0 and 100. The winner is the person who writes down the number that is going to be closer to 2/3 of the average of the number that everyone in class writes.

To win in this game one must consider what the rest of the class is going to do and act accordingly. The game is rather easily handled in a classroom situation. Three or four students in the class collect the papers from the students sitting close to them, and they add the numbers on the papers they collect in order to expedite the procedure of calculating the class average. We have played this game with the seniors for several years, and almost every time the winner is a person who writes a number close to 23. The winner is asked to explain their way of thinking, as are other students in the class. Everybody who pays attention in the class understands that if the numbers were written randomly, the average would have been 50. The winner usually thinks that 2/3 of 50 is 33, and thus, since almost everybody, the thinking continues, will write down a number close to 33, one needs to pick 2/3 of 33. That would be a number close to 22 or maybe a little higher than 22, in order to account for those who randomly write numbers without thinking through the problem. It is interesting, and contrary to economic theory, that the winner does not continue the thought process to assume that the rest of the class will reach the same conclusion. If that were the case, one would win if he or she would write down a number that is close to 2/3 of 22. If everybody thought this way, then one needs to pick 2/3 of that number, and after several iterations of this type of thinking, the "equilibrium point" according to economic theory is to pick the value of zero!

The goal of this experiment is to use a game theoretical approach to demonstrate to students how to better understand strategic decisions. A major point that can be made by the outcome of this game is that the decision should be made after considering what the other players are going to do and that the winner is the one who guesses how many iteration levels the competitors will consider.

This game serves as a good introduction to the problem of the winner's curse arising in common value auctions, which is relevant to engineers when, for example, they compete for design projects or for raw materials. In such cases, the value of the items is common but unknown during the bidding process (*e.g.*, bidding for exploration and production rights in a plot that one does not exactly know the quantity of oil and gas reserves). The winner's curse was first discussed by Capen, et al.,[3] a group of petroleum geologists who described the bidding outcome in offshore oil lease sales for the period 1954-1969. Studying the bids and profits of the companies participating in the auctions during this period, they observed that, "in a competitive oil and gas lease sale, or indeed in any bidding situation in which the ultimate value of the object to be won is subject to uncertainty, the highest bidder is the one who has overvalued the prize." In that sense, the winner is the most optimistic bidder, who is systematically overbidding (and losing money on average). This phenomenon was termed the winner's curse. It was caused by the failure of the bidders to use the optimal bidding strategy. The optimal bidding strategy should have taken into account what winning implies about the estimates of the competing firms. The winner's curse affected the ability of firms in the oil and gas industry to compete profitably in oil-lease sales and it is a phenomenon nested within many other applications (engineering contracts, etc.). More importantly, it is a wonderful way to explore with engineering students a very practical case where strategic decision making is crucial, and where methodologies now exist that can optimize a firm's behavior under uncertainty.

PRACTICAL ISSUES AND STUDENT FEEDBACK

The games presented here can be played either during a 1-hour and 15-minute class period or over two 45-minute class periods. We have used Games 1 and 2 in a class of participants in a workshop for Experimental Economics held at the University of William and Mary in May 2009, in a Master's-level graduate class of Managerial Economics in August 2009, and in a class of chemical engineering seniors in October 2009. In all cases, the spreadsheets were not available to the students long before the game, in order to avoid biased behavior. For the chemical engineers, the Excel files were e-mailed to the class about five minutes before class time.

The dice rolls can be done by digital means, either generating random numbers in Excel or using a dice-rolling website (*e.g.*, <http://www.random.org/dice/>). In fact, the chemical engineering seniors were so anxious to get to the dice rolls that they were using their own dice-rolling software on their laptops.

The feedback from the players included some ideas to make the games more fun or more relevant. Instead of making all the choices and then dice rolling 10 times, they suggested that dice rolling should follow after each one choice was made. In a larger class of about 45 students, however, it is not practical to go though this process. It was also suggested to adjust the value of the profits offered in the games to make them more relevant to the average student's income, instead of having profits on the order of millions or hundreds of thousands. Another suggestion was to reward the students according to their winnings, either with class credit or with monetary awards. One would expect that such a change in the format of the game, where the players would have a personal stake in the outcome, would lead to risk-averse behavior, which is consistent with economic theory. The goal of the games, however, is not to explore how the players react to different situations, but to illustrate to the players the concepts of risk and decision making under risk.

The response to an anonymous survey of whether participation in the games improved understanding of the utility function and of what is meant by attitude towards risk was that the games were helpful and that they should be incorporated in the class material for Design I. The spreadsheets, as well as the directions for conducting the games, are available to interested colleagues who might want to use them in their classes. The values of the options in the games can be changed according to the goals of each instructor.

CONCLUDING DISCUSSION

The advantage of running experiments in class, in addition to engaging students in active learning, is the ability to control external factors that may be affecting decision making as they change (*e.g.*, risk, uncertainty). Resources for designing other economics experiments are available on the web, for example through the Veconlab software developed by C. Holt at the University of Virginia[4] or through the EconPort portal developed by J. Cox, et al., at Georgia State University.[5]

Modern developments in economics and management include tools and techniques that address uncertainty, risk, strategic thinking, and decision making in a systematic and quantitative way. Deterministic models for the calculation of net present worth, for example, should be used to introduce the concept, but further analysis that incorporates financial uncertainty should be offered. The value of risk can be estimated with techniques like those presented by O'Donnell, et al.[1] The calculated risk can be used in conjunction with utility functions, such as those presented in the present work, to adjust the calculated NPW according to the methodology presented by Mansfield,[2] where the cash flows are substituted by their certainty equivalent values.

ACKNOWLEDGMENTS

The support of the National Science Foundation through a Course, Curriculum, and Laboratory Improvement grant (NSF-0737182) is gratefully acknowledged. An earlier version of this work has been presented at the AIChE meeting in Nashville in November 2009.

REFERENCES

1. O'Donnell, B.R., M.A. Hickner, and B.A. Barna, "Economic Risk Analysis, Using Analytical and Monte Carlo Techniques," *Chem. Eng. Ed.*, **36**(2), 94, (2002)
2. Mansfield, E., *Managerial Economics*, 4th Ed., W.W. Norton & Company Inc., New York (1999)
3. Capen, E.C., R.V. Clapp, and W.M. Campbell, "Competitive Bidding in High-Risk Situations," *J. Petrol. Tech.*, **23**, 641 (1971)
4. C. Holt, "Webgames and Strategic Behavior: Recipe for Interactive Learning," 2002, also: <http://veconlab.econ.virginia.edu/admin.htm>
5. Cox, J.C., and J.T. Swarthout, Georgia State University, EconPort web page, <http://www.econport.org/econport> ❐